TWO UNIT REDUNDANT SYSTEM WITH n-Failure Modes, n- Fault Detection and Inspection
Dr Sushant Pandey
Department Of Management,
Ishan Institute Of Management & Tecchnology
Gr. Noida

Abstract— The present paper gives the analysis of a two unit cold standby system with n failure modes of the online unit. The failed online unit is first sent for fault detection to decide its failure mode. After repair the unit is sent for inspection to decide whether the repaired unit is perfect or not. If it is found to be imperfect, then it is sent for post repair. Using regenerative points technique in Markov renewal process several effective measures of reliability are obtained.

Keywords— Laws - legislations - quality of life - sustainability.

I. INTRODUCTION
Many researchers working in the field of system reliability have analyzed two unit standby redundant system without considering the mode of failure of an online unit (operative unit). But in our daily life, we come across with engineering systems in which the online unit has more than one failure modes. Incorporating the idea of failure mode, we in, this paper analysed a two unit system with n-failure modes of the online unit. The failed unit is repaired, in the respective mode and after each repair it is inspected to decide whether the repaired unit is perfect or not. If the repaired unit is found to be imperfect then it is sent for post repair.

II. MODEL DESCRIPTION AND ASSUMPTIONS
(1) Consider a two unit system with one unit operative and the other as cold standby.
(2) Upon failure of an operative unit, the cold standby unit becomes operative instantaneously
(3) Before starting the repair, a failed unit is taken for its fault detection to decide the mode of failure. (4) After repair, the unit is inspected to decide whether the repaired unit is perfect or not. If the repair of a unit is found to be perfect then the repaired unit becomes operative or cold standby otherwise it is again sent for post repair. The probability of having perfect repair is fixed.
(5) Failure rate of an operative unit is constant, while the distributions of time for fault detection, repair, inspection and post-repair are general.
(6) A single repair facility is available for fault detection, repair, inspection and post-repair.
(7) Service discipline is FCFS.

Identifying the suitable regenerative points several measures of system effectiveness are discussed.

III. NOTATIONS AND STATES OF THE SYSTEM
N0 : Normal unit when it is operative
Ns : Normal unit when it is cold standby
Fwf : Failed unit waiting for fault detection
Ff : Failed unit under fault detection
FF : Fault detection is continued from earlier state
Frc : Failed unit in the Cth failure mode is under repair
Fic : Failed unit in the Cth failure mode is under inspection
Fpc : Failed unit in the Cth failure mode is under post repair
α : Constant failure rate of an operative unit.
F(,),F(.) : pdf and cdf of time to detect the mode of failure
Gc(,),Gc(.) : pdf and cdf of time for repair in the Cth failure mode.
Kc(,),Kc(.) : pdf and cdf of time for inspection in the Cth failure mode.
Hc(,),Hc(.) : pdf and cdf of time for post repair in the Cth failure mode.

P = (1-q) : Probability that the repair is perfect.
ml, m2c, m3c, m4c : Mean time for fault detection, repair inspection and post repair in the Cth failure mode.

Using the above notations, the possible states of the system are :

Up States :
So : (No, Ns) ; S1 : (No, Fr) ; S3c : (No, Frc)
S5c : (No, Fic) ; S7c : (No, Fpc)

Down States:
Se : (Fwf, FF) ; S4c : (Fwf, Frc) ; S6c : (Fwf, Fie) ; Sec : (Fwf, Fpc)

The possible transitions between the states are shown in Fig.1.

IV. TRANSITION AND STEADY STATE PROBABILITIES

Using the definition of $Q_{ij}(t)$ the transition probability matrix of the embedded Markov chain is

$$ P = (P_{ij} = [Q_{ii}(\infty)]_{ij} $$

with the non zero elements:

- $P_{1,3e} = f^*(\alpha)$
- $P_{3c,4e} = 1 - g^*(\alpha)$
- $P_{3c,5c} = g^*(\alpha)$
- $P_{4c,6c} = 1$
- $P_{5c,0} = PK^e(\alpha)$
- $P_{5c,6e} = 1 - K^e(\alpha)$
- $P_{5c,7c} = qK^e(\alpha)$
- $P_{6c,1} = P$
- $P_{6c,8c} = q$
- $P_{7c,8c} = 1-h^e(\alpha)$
- $P_{8c,1} = 1$
- $P_{1,4c} = 1-f^*(\alpha)$
- $P_{3c,6c} = 1 - p^e(\alpha)$

Thus we have

$$ P_{0} = \frac{1}{\alpha} $$
$$ P_{1} = \frac{1 - f^*(\alpha)}{\alpha} $$
$$ P_{2} = \frac{1}{\alpha} $$
$$ P_{3} = \frac{1 - g^*(\alpha)}{\alpha} $$
$$ P_{4} = \frac{1}{\alpha} $$
$$ P_{5} = \frac{1 - h^e(\alpha)}{\alpha} $$
$$ P_{6} = \frac{1}{\alpha} $$

V. MAN SOJOURN TIMES

As defined earlier, the mean sojourn time in state $S_{1e}$ is

$$ \mu_1 = E(T) = \int P[T > t] dt $$

Thus we have

$$ \mu_0 = \frac{1}{\alpha} $$
$$ \mu_3 = \frac{1 - g^*(\alpha)}{\alpha} $$
$$ \mu_4 = \frac{1}{\alpha} $$
$$ \mu_6 = \frac{1}{\alpha} $$
$$ \mu_7 = \frac{1 - h^e(\alpha)}{\alpha} $$
$$ \mu_8 = \frac{1}{\alpha} $$

VI. MEAN TIME TO SYSTEM FAILURE

To obtain the distribution function $11(t)$ of TSF with starting state $S_i$, we regard the failed states $S_2, S_4c, S_6c$ and $S_8c$ as absorbing.

Using the probabilistic arguments, the recursive relations among $\Pi_1(t)$ are:

$$ \Pi_0(0) = Q_{01}(0) \pi_1(0) $$
$$ \Pi_1(0) = Q_{12}(0) \sum Q_{13}(0) \pi_{13}(0) $$
$$ \Pi_3(0) = Q_{34}(0) + \sum Q_{35}(0) \pi_{53}(0) $$
$$ \Pi_5(0) = \sum Q_{56}(0) \phi_0(0) + \sum Q_{58}(0) \pi_{13}(0) $$
$$ \Pi_7(0) = \sum Q_{78}(0) + Q_{76}(0) \phi_0(0) $$

Taking Laplace-Stieltjes transform of these relations and solving for $\phi$ by omitting the argument "S" for brevity, therefore we get MTSF of the system when it starts operation from. So, as

$$ E(T) = \frac{-d MUST(0)}{d s} \bigg|_{s=0} = \frac{D^1(0) - N^1(0)}{D_1(0)} = N_1/D_1 $$

where

$$ N_1 = \mu_0 + \mu_1 + \sum \mu_{31c} (\mu_{3c} + \mu_{3c,5c} + \mu_{5c,7c} + \mu_{7c,3c,7c}) $$
$$ D_1 = 1 - \sum \mu_{31c} (\mu_{3c,5c} + \mu_{5c,7c} + \mu_{7c,0}) $$

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VII. AVAILABILITY ANALYSIS

As defined, the expression for $M_i(t)$ are

$$M_0(t) = e^{-(\alpha + \beta) t}$$ ; \hspace{1cm} M_1(t) = e^{-(\alpha + \beta) t}$$

$$M_3(t) = e^{-(\alpha + \beta) t} G_c(t)$$ ; \hspace{1cm} M_5(t) = e^{-(\alpha + \beta) t} K_c(t)$$

$$M_7(t) = e^{-(\alpha + \beta) t} H_c(t)$$

From the argument used in the theory of regenerative process, the pointwise $yAO)$ are seen to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_0(t) \neq A_1(t)$$

$$A_1(t) = M_1(t) + q_{1,3c}(t) \neq A_3(t) + \sum q_{1,4c}(t) \neq A_4(t)$$

$$A_3(t) = M_3(t) + \sum q_{3c,6c}(t) \neq A_6(t) + \sum q_{3c,5c}(t) \neq A_5(t)$$

$$A_4(t) = \sum q_{4c,6c}(t) \neq A_6(t)$$

$$A_5(t) = M_5(t) + q_{5e,0}(t) \neq A_0(t) + q_{5c,1}(t) \neq A_1(t) + \sum q_{5c,7c}(t) \neq A_7(t) + \sum q_{5c,8c}(t) \neq A_8(t)$$

$$A_6(t) = q_{6c,1}(t) \neq A_1(t) + \sum q_{6c,8c}(t) \neq A_8(t)$$

$$A_7(t) = M_7(t) + q_{7c,0}(t) \neq A_0(t) + q_{7c,1}(t) \neq A_1(t) + \sum q_{7c,5c}(t) \neq A_5(t) + \sum q_{7c,8c}(t) \neq A_8(t)$$

$$A_8(t) = q_{8c,1}(t) \neq A_1(t)$$

Taking Laplace-transform of the above equations and solving for $M_0(s)$ by omitting the argument "s" for brevity, therefore we (Jet the steady state availability when the system initially starts operation from $S_0$ is

$$A_0(s) = \lim_{s \to 0} sA_0(s) = N_2(0) / D_2(0) = N_2/D_2$$

VIII. BUSY PERIOD ANALYSIS

According to the definition of $W(t)$, we have

$$W_1 = \tilde{F}(t)$$ ; \hspace{1cm} $W_3(t) = \tilde{G}_c(t) - W_4(t)$

$$W_5(t) = \tilde{R}_c(t) - W_6(t)$$ ; \hspace{1cm} $W_7(t) = \tilde{A}_c(t) - W_8(t)$

Defining $B(t) = P[\text{system initial from regenerative state $S_1$ is under repair at epoch } t]$ the following relations among $131(t)$ hold good

$$B_0(t) = q_0(t) \neq B_1(t)$$

$$B_1(t) = W_1(t) + \sum q_{1,3c}(t) \neq B_3(t) + \sum q_{1,4c}(t) \neq B_4(t)$$

$$B_3(t) = W_3(t) + \sum q_{3c,5c}(t) \neq B_5(t) + \sum q_{3c,6c}(t) \neq B_6(t)$$

$$B_4(t) = W_4(t) + \sum q_{4c,6c}(t) \neq B_6(t)$$

$$B_5(t) = W_5(t) + q_{5c,0}(t) \neq B_0(t) + \sum q_{5c,1}(t) \neq B_1(t) + \sum q_{5c,7c}(t) \neq B_7(t) + \sum q_{5c,8c}(t) \neq B_8(t)$$

$$B_6(t) = W_6(t) + q_{6c,1}(t) \neq B_1(t) + \sum q_{6c,8c}(t) \neq B_8(t)$$

$$B_7(t) = W_7(t) + q_{7c,0}(t) \neq B_0(t) + \sum q_{7c,1}(t) \neq B_1(t) + \sum q_{7c,5c}(t) \neq B_5(t) + \sum q_{7c,8c}(t) \neq B_8(t)$$

$$B_8(t) = W_8(t) + q_{8c,1}(t) \neq B_1(t)$$

Taking Laplace-transform of the above equations and solving for $B(s)$ by omitting the argument "s" for brevity, one gets the fraction of time for which the system is under repair is given by
Using the definition of \( V_i(t) \) the following recursive relations among \( V_i(t) \) can be obtained:

\[
V_0(t) = Q_{00}(t) \times [1 + V_1(t)]
\]

\[
V_1(t) = \sum Q_{c0,0}(t) \times V_0(t) + Q_{1,4c}(t) \times V_4(t)
\]

\[
V_2(t) = \sum Q_{c0,0}(t) \times V_0(t) + \sum Q_{c,6c}(t) \times V_{6c}(t)
\]

\[
V_4(t) = \sum Q_{c,6c}(t) \times V_6(t)
\]

\[
V_5(t) = Q_{5c,0}(t) \times V_5(t) + Q_{bc,1}(t) \times V_1(t)
\]

\[
+ \sum Q_{c,7c}(t) \times V_7(t) + \sum Q_{c,8c}(t) \times V_{8c}(t)
\]

\[
V_7(t) = Q_{7c,0}(t) \times V_7(t) + Q_{7c,10}(t) \times V_{10}(t)
\]

\[
V_{8c}(t) = Q_{8c,1}(t) \times V_1(t)
\]

Taking Laplace-Stieltjes transform of the above equations and solving for \( V_0(s) \) by omitting the argument "s" for brevity, we get the expected number of visits per unit of time in steady state as:

\[
V_0 = \lim_{t \to \infty} \frac{V_0(t)}{t} = \lim_{s \to 0} \tilde{V}_0(s) = \frac{N_4}{D_2}
\]

where \( D_2 \) is same as in (53) and

\[
N_4 = \sum P_{1,3c} P_{3c,5c} (P_{5c,0} + P_{5c,7c} P_{7c,0})
\]

REFERENCES
