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PENTAGONAL HESITANT FUZZY MULTI-ATTRIBUTE DECISION MAKING BASED ON TOPSIS

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***Abstract*— Through this paper Pentagonal Hesitant Fuzzy Set is used to solve the higher order uncertainties with TOPSIS method. Pentagonal Hesitant Fuzzy Multi Attribute Decision Making model based on TOPSIS is developed and applied to the problems faced by the farmers who plant the rain fed crops in Villupuram District.**

Index Terms— Multi-Attribute Decision Making, Hesitant Fuzzy Set, Pentagonal Hesitant Fuzzy Set, Technique for Order Preference by Similarity to Ideal Solution, TOPSIS.

I. INTRODUCTION

Decision makers experience vagueness while dealing with real world problems. Such vagueness is solved by Fuzzy sets [19], introduced by Zadeh in the year 1965. Type-2 Fuzzy set [3], type-n Fuzzy Set, Intuitionistic Fuzzy Set, Fuzzy Multi-Set and Hesitant Fuzzy Set were the extensions resulting from fuzzy set. Though the last set was recognized recently, only the earlier four types had been applied to many life situations. Hesitant Fuzzy Set (HFS) [11,12] considers all decision making members and their values. Decisions making with the attributes are referred as Multi-Attribute Decision Making methods (MADM).

In classical Multi-Attribute Decision Making judgements are made properly with due ordering. Due to vagueness of human fantasy and the fuzzy nature, the objects of decision making are unquantifiable.

Sometimes false conclusions are arrived due to many limitations such as Expert's hesitation in defining the membership values, unfamiliarity over the attribute, lack of information about the problem concerned.

Hesitant Fuzzy Set (HFS) has been introduced by Torra [11] to motivate the Experts and decision makers in establishing membership degree of an element without any hesitation by choosing the right one. Hesitant situations are very common in various real world problems. Hence HFSs have attracted the attention of many researchers and this new approach facilitates the management of uncertainty motivated by hesitation. Fuzzy set accommodates on weaknesses and in turn produces a better solution when compared with MADM models.

II. BASIC DEFINITIONS

A. Fuzzy Set

Let E be the universal set, let x be an element of E, then $\underset{\sim}{A}$ of E is a set of ordered pairs

$$\underset{\sim}{A} = \{(x \mid \mu_{\underset{\sim}{A}}(x))\}, \text{ for all } x \in E \qquad (2.1)$$

where $\mu_{\underset{\sim}{A}}(x)$ is the grade (or) degree of membership of x in $\underset{\sim}{A}$.

$\mu_{\underset{\sim}{A}}(x)$ takes the value from the membership set M = [0,1] and

$\mu_{\underset{\sim}{A}}(x)$ is the membership function or characteristic function.

B. Hesitant Fuzzy Set

Let X be a fixed set, a Hesitant Fuzzy Set (HFS) on X is in terms of a function that when applied to X returns a subset of [0,1].

Mathematical representation of Hesitant fuzzy set [18]:

$$A = \{< x, {}_{H_A(X)} > / x \in X\} \qquad (2.2)$$

where $h_A(x)$ is a set of some values in [0,1], denoting the possible membership degrees of the element $x \in X$ to the set *A*.

C. Fuzzy Number

A Fuzzy number $\underset{\sim}{A}$ is a fuzzy set on the real line R, which must satisfy the following conditions.

(i) $\mu_{\underset{\sim}{A}}(x)$ is piecewise continous

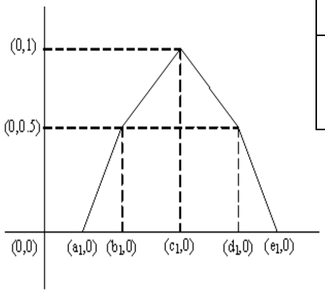
(ii) There exist atleast one $x_o \in R$ with $\mu_{\underset{\sim}{A}}(x_0) = 1$

(iii) $\underset{\sim}{A}$ must be normal and convex

D. Pntagonal Fuzzy Number

A Pentagonal Fuzzy Number (PFN) of a fuzzy set $\underset{\sim}{A}$ is defined as $\underset{\sim}{A}_P$ = {a, b, c, d, e}, and its membership function is given by,

$$\mu_{\underset{\sim}{A}_P}(x) = \begin{cases} 0 & for\ x < a \\ \frac{(x-a)}{(b-a)} & for\ a \le x \le b \\ \frac{(x-b)}{(c-b)} & for\ b \le x \le c \\ 1 & for\ x = c \\ \frac{(d-x)}{(d-c)} & for\ c \le x \le d \\ \frac{(e-x)}{(e-d)} & for\ d \le x \le e \\ 0 & for\ x > e \end{cases}$$



E. 2.5 Pentagonal Hesitant Fuzzy Set (PHFS)

Let X be a fixed set, then a pentagonal hesitant fuzzy set (PHFS) D on X is described as:

D = {< x ,P_t $h_A(x)$ >, x $\in$ X}

(2.3)

where P_t $h_A(x)$ is a set of Pentagonal fuzzy number expressing the possible membership and non membership degree of the element x $\in$ X to the set D.

III. Pentagonal Hesitant Fuzzy Element(PHFE)

If $h_A(x_i) = \{(a^L, a^{M_1}, a^{M_2}, a^{M_3}, a^U) / a^L \in h_A(x_i)$

then $h_A(x_i)$ is called Pentagonal Hesitant fuzzy element .

A. Hesitant Multiplicative Aggregation

To quantify the natural statements made by the decision maker, we employed Saaty's 1-9 scale with its respective meaning.

Table: The comparison between the 0.1-0.9 scale and the 1-9 scale

1-9 scale	**0.1-0.9 scale**	**Meaning**
1/9	0.1	Extremely not preferred
1/7	0.2	Very strongly not preferred
1/5	0.3	Strongly not preferred
1/3	0.4	Moderately not preferred
1	0.5	Equally preferred
3	0.6	Moderately not preferred
5	0.7	Strongly preferred
7	0.8	Very strongly preferred
9	0.9	Extremely preferred
Other values between 1 and 9 (2,4,6,8)	Other values between 0 and 1	Intermediate values used to present compromise

B. Pentagonal Hesitant Fuzzy Algorithm Based on TOPSIS:

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) is one of the well known classical MADM method which was first introduced by Hwang and Yoon. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution. The procedure for the FUZZY TOPSIS method as below

Step 1: Choose weight vector for each attribute according to their importance over the problem.

Step 2: Determine the corresponding pentagonal hesitant fuzzy positive ideal solution and pentagonal hesitant fuzzy negative ideal solution by the following expression;

$$A^{+} = \{<x_j, \max_i \{h_{ij}, g_{ij}\} > | \ j = 1,2,\ldots m\} \quad (3.1)$$

$$A^{-} = \{<x_j, \min_i \{h_{ij}, g_{ij}\} > | \ j = 1,2,\ldots m\} \quad (3.2)$$

Step 3: Calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the pentagonal hesitant fuzzy positive ideal A^+ and negative ideal A^- respectively by the following expression;

$$d_i^{+} = \{\textstyle\sum_{j=1}^{m} d(h_{ij}, h_j^{+}) w_j, \sum_{j=1}^{m} d(g_{ij}, g_j^{+}) w_j\} \quad (3.3)$$

$$d_i^{-} = \{\textstyle\sum_{j=1}^{m} d(h_{ij}, h_j^{-}) w_j, \sum_{j=1}^{m} d(g_{ij}, g_j^{-}) w_j\} \quad (3.4)$$

Step 4: Calculate the relative closeness to the ideal solution in the relative closeness of the alternative A_i with respect to d^+ is defined as:

$$c(A_i) = \frac{d_i^{-}}{d_i^{+} + d_i^{-}} \quad (3.5)$$

Step 5: Rank the preference order and the alternative with the largest relative closeness is chosen as the best alternative.

IV. Pentagonal Hesitant Fuzzy Set Application

Pathinathan.T and Johnson Savarimuthu.S [22] have been studying the problems faced by the farmers, who were planting the cash crops. In this paper we have extended our research work by analysing the rain fed cultivation in the same locality. It has been observed that the farmers of Villupuram district are planting rain fed crops like *Kambu,Cholam,Ulundudal,Thinai,Karamani* etc.

Rain fed cultivation depends on seasonal monsoon, water resources like rivers, tanks and irrigation wells. The water resources of this district have become dry. Even in borewells the water level have gone down so low. Farmers depend on electricity to pump water and electricity supply is very erratic. Hence farmers have to use diesel engines and as the cost of fuel goes up regularly beyond their debt increases.

A. Experts (from Villupuram District)

We collected the overall information regarding agriculture problems from the following five Experts in Villupuram District.

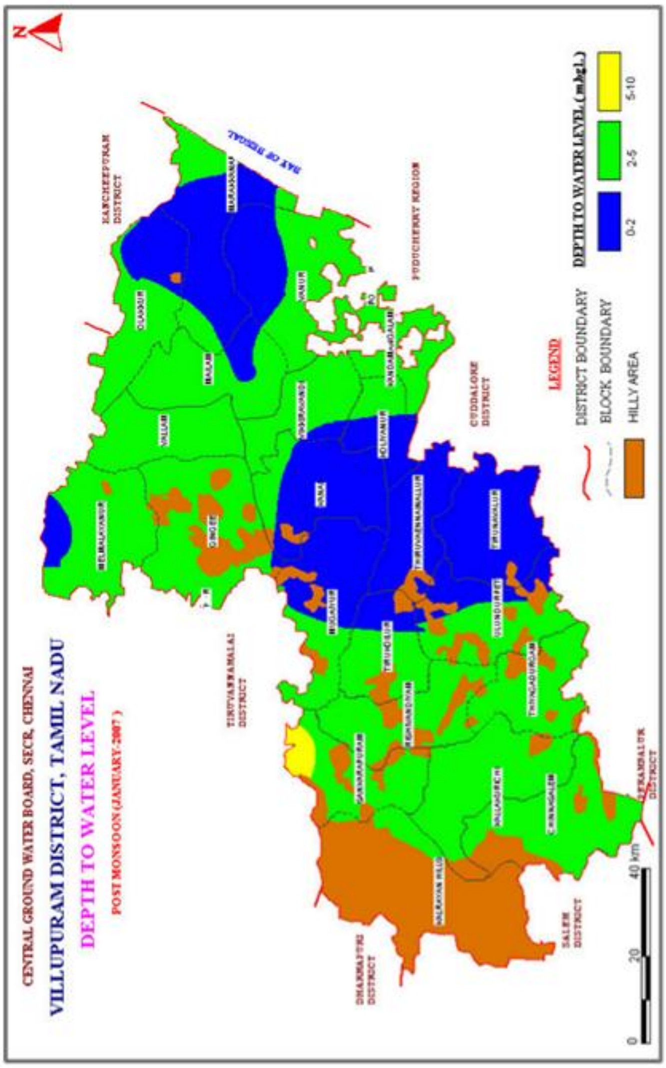
DM_1. Mr. P. Ravi, farmer who owns a land, Kallakurichi.

DM_2. Mr.S.Rajesh, farmer who is doing share cropping (Kuthagai), Sankarapuram.

DM_3. Mr.Arockiaraj, Private Agricultural Officer, Viriyur.

DM_4. Mr. A. Anand, Moongilthuraipattu

DM_5. Mr.Kirubanithu, Farmer, Thiyaga Durgan



B. Alternatives

The following are the major rain fed crops which are cultivated in Villupuram district. We took these crops as our alternatives.

A_1. *Kambu* (Millet)

A_2. *Cholam* (Maize)

A_3. *Ulundudal* (Black gram)

A_4. *Thinai* (Fox tail millet)

A_5. *Karamani* (Black eyed bean)

C. Attributes

We collected a few major problems faced by the farmers and we interviewed them to filter out four major attributes .

(i) Benefit Type (Qualitative in nature)

(ii) Cost Type (Quantitative in nature)

X_1. Crop failure (Benefit Type) – Due to reduction in crop yield, nutritional need of the people is not met. Failure of the crop means drying crop and inability to save the standing crop.

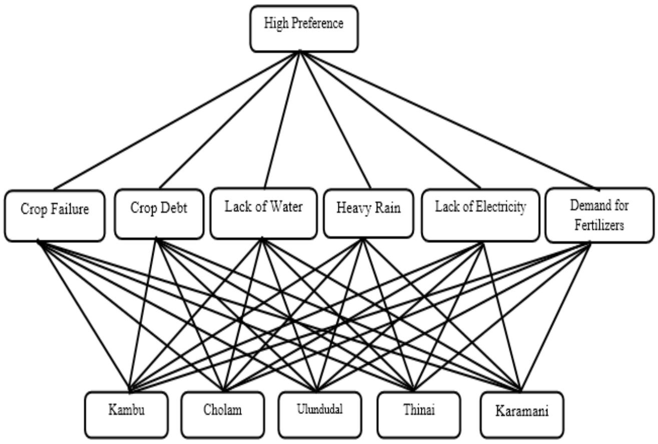
X_2. Crop debt (Cost Type) – Borrowing of money from private money lenders, agriculture debt to meet the expenses increases the interest.

X_3. Lack of water (Benefit Type) - Water scarcity in Gamukha, Sathanur Dam and truant monsoon.

X_4. Heavy rain and Cyclone (*Nilam*) (Benefit Type) – Soil erosion, loss of soil fertility is caused; livelihood and farming lands have been destroyed by *Nilam* cyclone in the recent year across the Villupuram District.

D. Hierarchical Structure

The hierarchical structure of this decision making problem is shown from the below diagram;



V. Adaptation and Description of the problem

Hesitant fuzzy decision matrix is obtained by considering each and every experts' opinion with their possible membership values and they are recorded as follows:-

Table 5.1: Hesitant Fuzzy Decision Matrix (by utilizing Step 1 in Algorithm)

Table 5.1: Hesitant Fuzzy Decision Matrix (by utilizing Step 1 in Algorithm)

	X_1	X_2	X_3	X_4
A_1	(0.6,0.4,0.7,0.3,0.5)	(0.4,0.5,0.6,0.2,0.3)	(0.7,0.6,0.8,0.5,0.3)	(0.5,0.8,0.6,0.5,0.3)
A_2	(0.3,0.5,-,-,0.6)	(0.7,0.6,0.5,0.2,0.3)	(0.8,0.7,0.6,0.5,0.3)	(0.5,-,0.7,0.5,0.5)
A_3	(-,-,0.5,-,0.4)	(0.6,0.7,0.5,0.2,0.3)	(-,0.6,0.5,0.4,0.2)	(0.8,-,0.7,0.5,0.2)
A_4	(0.3,-,0.6,-,0.5)	(0.8,0.7,0.6,0.3,0.4)	(-,-,0.5,0.4,0.2)	(-,-,0.7,0.5,0,3)
A_5	(0.5,0.3,0.9,0.6,0.3)	(0.8,0.7,0.4,0.1,0.2)	(0.9,-,0.6,0.4,0.2)	(0.3,0.8,0.6,0.4,0.2)

where,

A_i, i = 1,2,3,4,5 (five alternatives); X_i ,i = 1,2,3,4 (four attributes) and $A_1(X_i)$, i = 1,2,3,4,5 denotes Alternative 1 (Millet) comparing with all four attributes. And all the five experts are asked to give their opinions and the opinions are tabulated.

$A_1(X_i) = (PT_1, PT_2, PT_3, PT_4)$

For instance, $A_1(X_1)$ = (0.6,0.4,0.7,0.3,0.5) denotes on discussing Alternative 1 (Millet) with attribute 1 (crop failure), Decision Maker 1 (PT_1) provide 0.1 as the membership value, indicates crop failure considered to be the biggest burden for farmer who involve in Millet cultivation.

Similarly, PT_2 provide 0.4 as the membership value and so on…

Suppose, if $A_2(X_1)$ = (0.3,0.5, -, -, 0.6), denotes on discussing Alternative 2 (Maize) with the attribute 1 (crop failure), PT_3 and PT_4 failed to record their values due to the lack of knowledge about the respective alternative over the attribute.

Table 5.2: Hesitant Fuzzy Decision Matrix

	X_1	X_2	X_3	X_4
A_1	{0.6,0.4,0.7,0.3,0.5}	{0.4,0.5,0.6,0.2,0.3}	{0.7,0.6,0.8,0.5,0.3}	{0.5,0.8,0.6,0.5,0.3}
A_2	{0.3,0.5,0.3,0.3,0.6}	{0.7,0.6,0.5,0.2,0.3}	{0.8,0.7,0.6,0.5,0.3 }	{0.5,0.4,0.7,0.5,0.4 }
A_3	{0.4,0.4,0.5,0.4,0.4}	{0.6,0.7,0.5,0.2,0.3}	{0.2,0.6,0.5,0.4,0.2}	{0.8,0.2,0.7,0.5,0.2}
A_4	{0.3,0.3,0.6,0.3,0.5}	{0.8,0.7,0.6,0.3,0.4}	{0.2,0.2,0.5,0.4,0.2}	{0.3,0.3,0.7,0.5,0.3}
A_5	{0.5,0.3,0.7,0.6,0.4}	{0.7,0.6,0.5,0.2,0.3}	{0.7,0.3,0.6,0.4,0.3}	{0.3,0.8,0.6,0.4,0.2}

Table 5.3: Pentagonal Hesitant Fuzzy Decision Matrix

	X_1	X_2	X_3	X_4
A_1	{0.4,0.5,0.6,0.7,0.8} {0.2,0.3,0.4,0.5,0.6} {0.5,0.6,0.7,0.8,0.9} {0.1,0.2,0.3,0.4,0.5} {0.3,0.4,0.5,0.6,0.7}	{0.4,0.5,0.6,0.7,0.8} {0.3,0.4,0.5,0.6,0.7} {0.4,0.5,0.6,0.7,0.8} {0.1,0.2,0.3,0.4,0.5} {0.1,0.2,0.3,0.4,0.5}	{0.5,0.6,0.7,0.8,0.9} {0.4,0.5,0.6,0.7,0.8} {0.6,0.7,0.8,0.9,0.9} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5}	{0.3,0.4,0.5,0.6,0.7} {0.6,0.7,0.8,0.9,0.9} {0.4,0.5,0.6,0.7,0.8} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5}
A_2	{0.1,0.2,0.3,0.4,0.5} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5} {0.1,0.2,0.3,0.4,0.5} {0.4,0.5,0.6,0.7,0.8}	{0.5,0.6,0.7,0.8,0.9} {0.4,0.5,0.6,0.7,0.8} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5} {0.1,0.2,0.3,0.4,0.5}	{0.6,0.7,0.8,0.9,0.9} {0.5,0.6,0.7,0.8,0.9} {0.4,0.5,0.6,0.7,0.8} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5}	{0.3,0.4,0.5,0.6,0.7} {0.2,0.3,0.4,0.5,0.6} {0.4,0.5,0.6,0.7,0.8} {0.3,0.4,0.5,0.6,0.7} {0.2,0.3,0.4,0.5,0.6}
A_3	{0.2,0.3,0.4,0.5,0.6} {0.2,0.3,0.4,0.5,0.6} {0.3,0.4,0.5,0.6,0.7} {0.2,0.3,0.4,0.5,0.6} {0.2,0.3,0.4,0.5,0.6}	{0.4,0.5,0.6,0.7,0.8} {0.5,0.6,0.7,0.8,0.9} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5} {0.3,0.4,0.5,0.6,0.7}	{0.1,0.2,0.3,0.4,0.5} {0.4,0.5,0.6,0.7,0.8} {0.3,0.4,0.5,0.6,0.7} {0.2,0.3,0.4,0.5,0.6} {0.1,0.2,0.3,0.4,0.5}	{0.1,0.2,0.3,0.4,0.5} {0.1,0.2,0.3,0.4,0.5} {0.5,0.6,0.7,0.8,0.9} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5}
A_4	{0.1,0.2,0.3,0.4,0.5} {0.1,0.2,0.3,0.4,0.5} {0.4,0.5,0.6,0.7,0.8} {0.1,0.2,0.3,0.4,0.5} {0.3,0.4,0.5,0.6,0.7}	{0.6,0.7,0.8,0.9,0.9} {0.5,0.6,0.7,0.8,0.9} {0.4,0.5,0.6,0.7,0.8} {0.1,0.2,0.3,0.4,0.5} {0.3,0.4,0.5,0.6,0.7}	{0.1,0.2,0.3,0.4,0.5} {0.1,0.2,0.3,0.4,0.5} {0.3,0.4,0.5,0.6,0.7} {0.2,0.3,0.4,0.5,0.6} {0.1,0.2,0.3,0.4,0.5}	{0.1,0.2,0.3,0.4,0.5} {0.1,0.2,0.3,0.4,0.5} {0.5,0.6,0.7,0.8,0.9} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5}
A_5	{0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5} {0.5,0.6,0.7,0.8,0.9} {0.4,0.5,0.6,0.7,0.8} {0.2,0.3,0.4,0.5,0.6}	{0.5,0.6,0.7,0.8,0.9} {0.4,0.5,0.6,0.7,0.8} {0.3,0.4,0.5,0.6,0.7} {0.1,0.2,0.3,0.4,0.5} {0.1,0.2,0.3,0.4,0.5}	{0.5,0.6,0.7,0.8,0.9} {0.1,0.2,0.3,0.4,0.5} {0.4,0.5,0.6,0.7,0.8} {0.2,0.3,0.4,0.5,0.6} {0.1,0.2,0.3,0.4,0.5}	{0.1,0.2,0.3,0.4,0.5} {0.6,0.7,0.8,0.9,0.9} {0.4,0.5,0.6,0.7,0.8} {0.2,0.3,0.4,0.5,0.6} {0.1,0.2,0.3,0.4,0.5}

By adapting the algorithm given in the section 3 and by using the equation (3.1and 3.2), we have the following positive ideal and negative ideal solution for each alternative over the attribute.

Table 5.3: Positive Ideal Solution (by Eqn: 3.1)

	Positive Ideal Solution		Negative Ideal Solution
A_1^+	$\{<x_1: 0.5,0.6,0.7,0.8,0.9>\}$	A_1^-	$\{<x_1: 0.1,0.2,0.3,0.4,0.5>\}$
A_2^+	$\{<x_2: 0.6,0.7,0.8,0.9,0.9>\}$	A_2^-	$\{<x_2: 0.1,0.2,0.3,0.4,0.5>\}$
A_3^+	$\{<x_3: 0.6,0.7,0.8,0.9,0.9>\}$	A_3^-	$\{<x_3: 0.1,0.2,0.3,0.4,0.5>\}$
A_4^+	$\{<x_4: 0.6,0.7,0.8,0.9,0.9>\}$	A_4^-	$\{<x_4: 0.1,0.2,0.3,0.4,0.5>\}$

Then, by utilizing the equation (3.3 and 3.4), we have the following distance values;

Hamming Distance Calculations

$$d_1^+ = 0.25\sqrt{\frac{|0.1-0.5|^2+|0.3-0.6|^2+|0.3-0.7|^2+|0.3-0.8|^2+|0.4-0.9|^2}{5}} + 0.25\sqrt{\frac{|0.4-0.6|^2+|0.5-0.7|^2+|0.5-0.8|^2+|0.2-0.9|^2+|0.3-0.9|^2}{5}} + 0.35\sqrt{\frac{|0.2-0.6|^2+|0.2-0.7|^2+|0.5-0.8|^2+|0.4-0.9|^2+|0.2-0.9|^2}{5}} + 0.15\sqrt{\frac{|0.3-0.6|^2+|0.2-0.7|^2+|0.6-0.8|^2+|0.4-0.9|^2+|0.2-0.9|^2}{5}}$$

$$= 0.465$$

Table 5.4: Hamming Distance for HFS (by Eqn: 3.3)

d_i^+	P-Distance	d_i^-	N-Distance
d_1^+	0.465	d_1^-	0.36
d_2^+	0.45	d_2^-	0.365
d_3^+	0.43	d_3^-	0.395
d_4^+	0.465	d_4^-	0.37
d_5^+	0.465	d_5^-	0.49

By using equation 3.5, we recorded the closeness values among the alternatives and they are tabulated as follows;

Table 5.5: Closeness for HFS (by Eqn: 3.5)

	d_i^-	$d_i^+ + d_i^-$	$c(A_i)$
$c(A_1)$	0.36	0.825	0.4363
$c(A_2)$	0.365	0.815	0.4478
$c(A_3)$	0.395	0.825	0.4787
$c(A_4)$	0.37	0.835	0.4431
$c(A_5)$	0.49	0.955	0.5130

From the above table we rank the alternatives A_i (i = 1,2,3,4,5) as:-

$A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$

VI. Conclusion

Aggregating the opinion from the five Decision makers, we have the preference ranking order relation as $A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$, (i.e.,) Alternative A_5 (Black eyed bean) is dominated by all the other alternatives. Black gram (A_3) and Maize (A_2) almost share the same ranking position when compared with other alternatives. Millet and Fox tail millet are the two crops that provide some relief to the farmer's struggle.

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