

IMAGE RECONSTRUCT USING COMPRESSIVE SENSING

¹Akanksha Thapliyal

¹Graphic Era University Dehradun, Department Of Computer Science
akansh20.anki@gmail.com

Abstract— A sparse signal in a high dimensional space, compressive sensing system, which combines with sampling and compression, can reconstruct that signal accurately and efficiently from fewer linear measurements much less than its actual dimension using sparse priors of signal. Currently, researchers always use orthogonal wavelet to represent the images. But the wavelet only has single scaling function and can not simultaneously satisfy the orthogonality, high vanishing moments, compact support, symmetry characteristic and regularity. Developed from the theory of wavelet, multi-wavelet transform, which can simultaneously satisfy the five characteristics, provides a great potential to obtain high-performance coding. According to the three main steps (Sparse representation, measurement matrix, reconstruction algorithm) of compressive sensing image reconstruction, this paper proposes a compressive sensing image reconstruction based on sparse representation of the image in multi-wavelet transform domain while using Orthogonal Matching Pursuit iterative as the reconstruction algorithm. The experimental results show that the reconstructed image has better vision quality and a good performance on PSNR. Meanwhile, the algorithm of reconstruction gets a faster convergence rate.

Index terms- Compressed Sensing, Kalman Fied Compressed Sensing, dynamic MRI,

I. INTRODUCTION

In recent work, the problem of causally reconstructing time sequences of spatially sparse signals, with unknown and slow time varying sparsity patterns, from a limited number of linear “incoherent” measurements was studied and a solution called Compressed Sensing (CS) was proposed. An important example of this type of problems is real-time medical image sequence reconstruction using MRI, for e.g. dynamic MRI to image the beating heart or functional MRI to image the brain’s neuronal responses to changing stimuli(see Fig.1). In these examples, the signal (heart or brain image) is approximately sparse (compressible) in the wavelet transform domain. MRI measures the 2D Fourier transform of the image which is known to be “incoherent” w.r.t. the wavelet basis. Because MR data acquisition is sequential, the scan time (time to get enough data to accurately reconstruct one frame) is reduced if fewer measurements are needed for accurate reconstruction and hence there has been a lot of interest in the MRI community to use compressed sensing (CS) to do this.

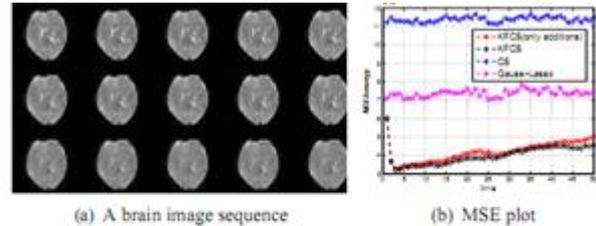


Fig. 1. The three rows of Fig.1(a) show some frames of a brain image sequence(row 1), its MRI reconstructions using KF-CS(row 2), and using CS(row 3). Fig.1(b) is the corresponding MSE plot. $n = 2049$, $m = 4096$ and $\sigma_{obs}^2 = 100$.

This idea is first demonstrated for a single MR image or volume. The work of extending the idea to offline dynamic MRI reconstruction, i.e. it used the entire time sequence of measurements to jointly estimate the entire image sequence (treated it as a 3D x-yt signal, sparse in wavelet domain along the x-y axis and sparse in the Fourier domain along the time axis). But this is a batch solution (needs all measurements first) and also the resulting joint optimization is computationally complex. On the other hand, the solution is causal and also much faster, and thus can be used to make dynamic MRI real-time. Reduced scan-time and real-time reconstruction are the currently missing abilities that prevent the use of MRI in interventional radiology applications, such as MR-guided surgery.

In this work we use to develop a CS algorithm to causally reconstruct image sequences using MR data. There are some key differences in our current problem from the simplistic model used and these require some practical modifications to the algorithm of CS. Additionally, in this work, (i) we develop a method for estimating the prior model parameters from training data and (ii) we use the results to develop a method for selecting the number of observations required and the parameters used by the CS step. Results on reconstructing a cardiac sequence and a brain sequence show greatly reduced mean squared error(MSE) when compared to performing CS at each time, as well as to some other modifications of CS. For e.g. in Fig 1b, the CS error is more twice that of filtered CS.

1.1 Problem Formulation

Let $(Z_t)_{m1 \times m2}$ denote the time at t and let $m := m1 \times m2$ be its dimension. Let X_t denote the 2D discrete wavelet transform(DWT) of Z_t , i.e. $X_t := WZ_tW'$. Let F denote the discrete Fourier Transform (DFT) matrix and $Y_{full,t} :=$

$FZ_tF' = FW_tX_tW_tF'$ denote the 2D-DFT of Z_t . All of this can be transformed to a 1D problem by using Kronecker product. Let $y_{full,t} := \text{vec}(Y_{full,t})$ and $x_t := \text{vec}(X_t)$. Then $y_{full,t} = F_{1D}W_{1D}x_t$ where $F_{1D} = \text{Kronecker product of } F \text{ and } W_{1D} = \text{Kronecker product of } W$. Here, $\text{vec}(X_t)$ denotes the vectorization of the matrix X_t formed by stacking the columns of X_t into a single column vector. In MR imaging, we capture a set of n , ($n < m$), Fourier coefficients corrupted by white noise. This can be modeled by applying a $n \times m$ mask, M (which contains a single 1 at a different location in each row and all other entries are zero) to $y_{full,t}$ followed by adding Gaussian noise. The above can be rewritten using the notation

II. COMPRESSIVE SENSING

CS relies on two principles: sparsity, which pertains to the properties of natural signals of interest, and incoherence, which involves how signal is sensed/sampled. The basic principle is that sparse or compressible signals can be reconstructed from a surprisingly small number of linear measurements, provided that the measurements satisfy an incoherence property. Such measurements can then be regarded as a compression of the original signal, which can be recovered if it is sufficiently compressible.

Sparsity:

In particular, many signals are sparse, that is, they contain many coefficients close to or equal to zero, when represented in some domain.

Incoherence:

Incoherence extends the duality between time and frequency. It expresses the idea that objects having a sparse representation in Ψ must be spread out in the domain in which they are acquired. This is similar to the analogy in which Dirac or a spike in the time domain is spread out in the frequency domain. Incoherence is necessary for acquiring good linear measurement in the new measurement space.

STEPS OF COMPRESSIVE SENSING

1) Select an appropriate wavelet function and set a required decomposition level, then execute the wavelet packet foil decomposition on the original image.

2) Determine the optimal basis of the wavelet packet in the light of the Shannon entropy criterion.

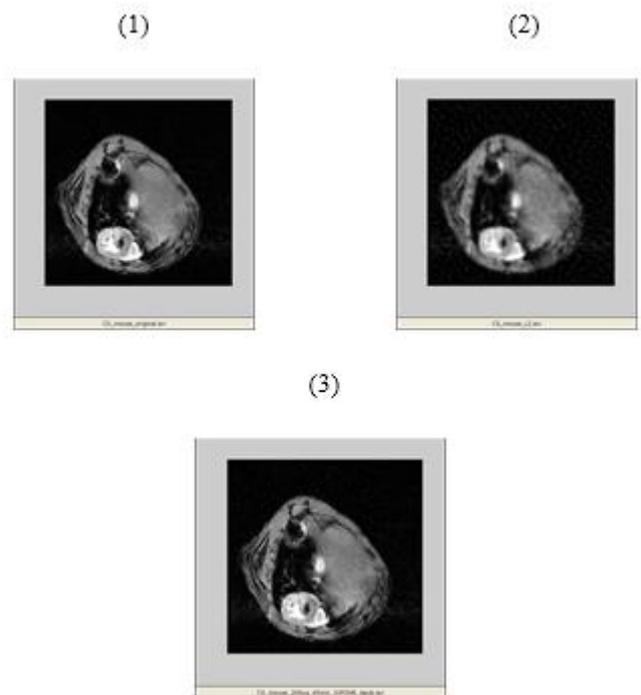
3) As the main information and energy of the original image are concentrated in the low frequency subband by the wavelet packet transform, which plays a very important role in the image reconstruction, all the low-frequency coefficients are compressed losslessly in order to reduce the loss of the useful information.

4) According to the theory of CS, select an appropriate random measurement matrix, and make measurement encoding on all the high frequency coefficients in line with the optimal

basis of the wavelet packet, and obtain the measured coefficients.

5) Restore all the high-frequency coefficients with the method of OMP from the measured coefficients.

6) Implement the wavelet packet inverse transform to all the restored low-frequency and high frequency coefficients, and reconstruct the original image.



Compressed Sensing in Dynamic MRI

These movies demonstrate the power of compressed sensing for rapid acquisition in dynamic MRI. (1) Shows an image sequence of a mouse heart beating that has been densely sampled. Were we to attempt to sub-sample it say by a factor of 5 (20% of Nyquist) and use linear reconstruction techniques we would experience large qualities of aliasing interference (2). However using nonlinear reconstruction algorithms, in this case our recently developed [Stage wise Conjugate Gradient Pursuit algorithm](#), we can generate a good reconstruction without aliasing, (3).

III. IMAGE RECONSTRUCT METHOD

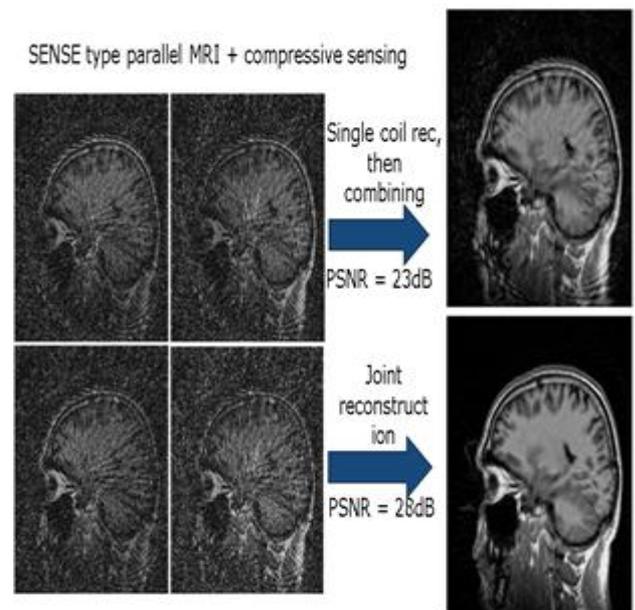
Image reconstruction in CT is a mathematical process that generates images from X-ray projection data acquired at many different angles around the patient. Image reconstruction has a fundamental impact on image quality and therefore on radiation dose. For a given radiation dose it is desirable to reconstruct images with the lowest possible noise without sacrificing image accuracy and spatial resolution. Reconstructions that improve image quality can be translated into a reduction of radiation dose because images of acceptable quality can be reconstructed at lower dose.

Two major categories of methods exist, analytical reconstruction and iterative reconstruction. Methods based on filtered backprojection (FBP) are one type of analytical reconstruction that is currently widely used on clinical CT scanners because of their computational efficiency and numerical stability. Many FBP-based methods have been developed for different generations of CT data-acquisition geometries, from axial parallel- and fan-beam CT in the 1970s and 1980s to current multi-slice helical CT and cone-beam CT with large area detectors. For a general introduction of the fundamental principles of CT image reconstruction, please refer to Chapter 3 in Kak and Slaney's book (1). An introduction to reconstruction methods in helical and multi-slice CT can be found in Chapters 9 and 10 in Hsieh's book (2). A review of CT image reconstruction methods used on clinical CT scanners can be found in the article by Flohr, et al (3).

Users of clinical CT scanners usually have very limited control over the inner workings of the reconstruction method and are confined principally to adjusting various parameters specific to different clinical applications. The reconstruction kernel, also referred to as "filter" or "algorithm" by some CT vendors, is one of the most important parameters that affect the image quality. Generally speaking, there is a tradeoff between spatial resolution and noise for each kernel. A smooth kernel generates images with lower noise but with reduced spatial resolution. A sharp kernel generates images with higher spatial resolution, but increases the image noise.

The selection of reconstruction kernel should be based on specific clinical applications. For example, smooth kernels are usually used in brain exams or liver tumor assessment to reduce image noise and enhance low contrast detectability. Radiation dose associated with these exams is usually higher than that for other exams due to the intrinsic lower contrast between tissues. On the other hand, sharper kernels are usually used in exams to assess bony structures due to the clinical requirement of better spatial resolution. Lower radiation dose can be used in these exams due to the inherent high contrast of the structures.

Another important reconstruction parameter is slice thickness, which controls the spatial resolution in the longitudinal direction, influencing the tradeoffs among resolution, noise, and radiation dose. It is the responsibility of CT users to select the most appropriate reconstruction kernel and slice thickness for each clinical application so that the radiation dose can be minimized consistent with the image quality needed for the examination.



In addition to the conventional reconstruction kernels applied during image reconstruction, many noise reduction techniques, operating on image or projection data, are also available on commercial scanners or as third-party products. Many of these methods involve non-linear de-noising filters, some of which have been combined into the reconstruction kernels for the users' convenience. In some applications these methods perform quite well to reduce image noise while maintaining high-contrast resolution. If applied too aggressively, however, they tend to change the noise texture and sacrifice the low-contrast detectability in the image. Therefore, careful evaluation of these filters should be performed for each diagnostic task before they are deployed into wide-scale clinical usage. Scanning techniques and image reconstructions in ECG-gated cardiac CT have a unique impact on image quality and radiation dose. Half-scan reconstruction is typically used to obtain better temporal resolution. For the most widely employed retrospectively ECG-gated helical scan mode, the helical pitch is very low (~0.2 to 0.3) in order to avoid anatomical discontinuities between contiguous heart cycles. A significant dose reduction technique in helical cardiac scanning is ECG tube-current pulsing, which involves modulating the tube current down to 4% to 20% of the full tube current for phases that are of minimal interest. Prospectively ECG-triggered sequential (or step-and-shoot) scans are a more dose-efficient scanning mode for cardiac CT, especially for single-phase studies. An overview of scanning and reconstruction techniques in cardiac CT can be found in an article by Flohr et al (4).

Iterative reconstruction has recently received much attention in CT because it has many advantages compared with conventional FBP techniques. Important physical factors including focal spot and detector geometry, photon statistics, X-ray beam spectrum, and scattering can be more accurately incorporated into iterative reconstruction, yielding lower image noise and higher spatial resolution compared with FBP (5). In addition, iterative reconstruction can reduce image artifacts such as beam hardening, windmill, and metal artifacts. A recent

clinical study on an early version of iterative reconstruction demonstrated a potential dose reduction of up to 65% (6) compared with FBP-based reconstruction algorithms. Due to the intrinsic difference in data handling between FBP and iterative reconstruction, images from iterative reconstruction may have a different appearance (e.g., noise texture) from those using FBP reconstruction. Careful clinical evaluation and reconstruction parameter optimization will be required before iterative reconstruction can be accepted into mainstream clinical practice. High computation load has always been the greatest challenge for iterative reconstruction and has impeded its use in clinical CT imaging. Software and hardware methods are being investigated to accelerate iterative reconstruction. With further advances in computational technology, iterative reconstruction may be incorporated into routine clinical practice in the future.

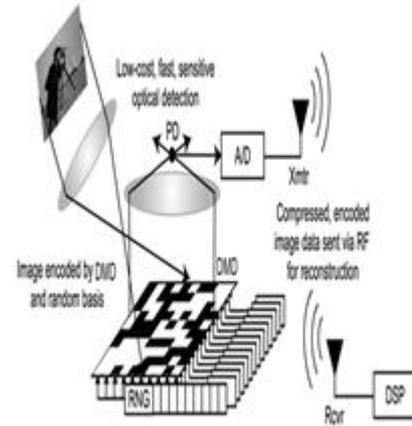
OMP Algorithm:

Image reconstruction algorithm based on compressed sensing using conjugate gradient is proposed for the first time in this paper. Compressed sensing is a technique for acquiring and reconstructing a signal or image utilizing the prior knowledge that is sparse or compressible. During the past several decades scholars have made all sorts of guesses about the prior $Pr(x)$ for images in order to find its sparse representation and also proposed some available algorithms like matching pursuit (MP) and orthogonal matching pursuit (OMP) algorithms. Some reconstruction algorithms used the convex relaxation method, but the conjugate gradient is a method with simpler iterative process and less memory requirement compared with the least square and Newton iteration. Simulation results show that this image reconstruction algorithm based on compressed sensing using conjugate gradient gets better performance on time and PSNR than OMP algorithm.

Input: The measurement y and measurement matrix A

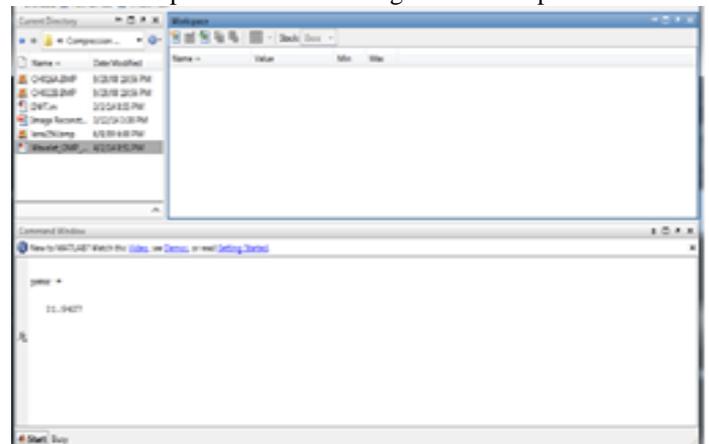
Output: Reconstructed signal x^*

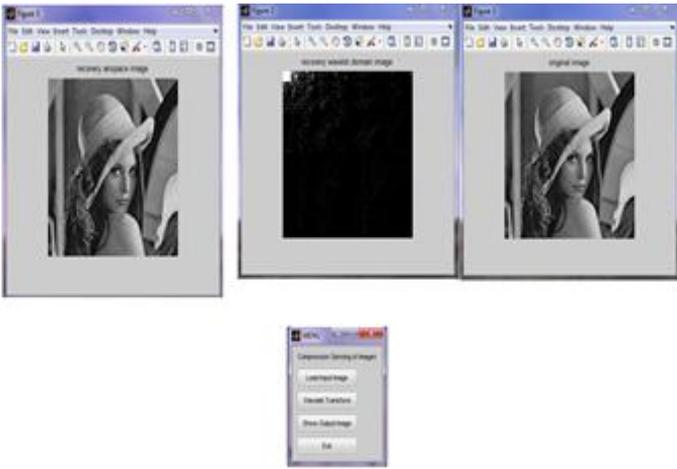
- (1) Initialize the residual $r_0 = y$, index set $C_0 = \#$ and counter $k=1$.
- (2) Find the column vector a_{ck} of A that is mostly correlated with the residual:
 $ck = \text{argmax}_c |hr_{k-1}, a_{ci}|, c \# [n]$
 $C_k = C_{k-1} \cup \{ck\}$
- (3) Solve the least-square problem:
 $x_k = \text{argmin}_x \|y - AC_k x\|_2$
 where AC_k denotes the columns of A indexed by C_k .
- (4) Update the residual to remove the contribution of a_{ck}
 $r_k = y - AC_k x_k$
- (5) Increment k , and go back to step (2) until stopping criterion holds.
- (6) Return the output x^* with $x^*(i) = x_k(i)$ for $i \# C_k$



IV. SIMULATION AND RESULT

To gain some insights into the the effect of the proposed SMM-OMP on sparse signal recovery, we evaluate 10000 independent Monte-Carlo trails, the nonzero variables of sparse signal β are generated randomly from a Gaussian distribution and subject to $[\beta]^2 = 1$. the signal length is set to $N=48$ and the number of measurements are set from 16 to 40. The position of nonzero variables of β are generated randomly. Consider the signal to noise ratio (SNR) as $SNR=10\text{dB}$. We observe that the SMM-OMP has a better recovery performance than RMM-OMP on sparse signal recovery problem, OMP algorithm can obtain a better performance if the signal is more sparser.





V. CONCLUSION

In this paper, we have developed the filtered CS idea for causal reconstruction of medical image sequences from MR data and have shown greatly improved reconstruction results on MRI data, as compared to CS and its modifications. This is the first real application of filtered CS and is considerably more difficult than simulation data because the measurement matrix for MR is not as incoherent as a random Gaussian matrix and because the different wavelet coefficients have vastly different magnitudes and variances. Future work will involve a rigorous

analysis of the proposed algorithmic ideas and using it to propose a novel filtered CS based algorithm for compressible sequences. compressive sensing technique.

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