ESTIMATION OF CHANNELS IN OFDM EMPLOYING CYCLIC PREFIX

Manisha Mohite
Department Of Electronics and Telecommunication
Terna College of Engineering, Nerul,
Navi-Mumbai, India
manisha.vhantale@gmail.com

Abstract— Modern communications system needs to be based upon (OFDM).

The channel estimation can be appropriate to provide for analyzing effect of the channel on signal by either inserting pilot tones into all of the subcarriers of OFDM symbols with a specified period or inserting pilot tones into individual OFDM symbols. The block type pilot channel estimation develops for slow fading channels and comb type for fast fading. When the data transmits at high bit rates, the channel impulse response h(t) can extend over symbol periods, it leads to ISI. OFDM is committed as part of the appropriate factors to mitigate the ISI.

This work improves various channels performance measures based on the comparison of various channel estimation algorithms and suggests a novel technique which provides better performance. This paper discusses the estimations of channels in OFDM and its implementations in MATLAB using a cyclic prefix and pilot based block type channel estimation techniques by linear square and minimum mean square estimator algorithms.

Index Terms— OFDM, Channel Estimation, LS (linear square), MMSE (minimum mean square estimator).

I. INTRODUCTION

Wireless communication system demands to the higher data rates with low delays and low BER. In such condition, the wireless channel situation mainly governs the performance of wireless communication systems. Generally, high data rate transmission and high mobility of transmitters and receivers usually result in the frequency-selective and time-selective, i.e. time varying, fading channels for future mobile broadband wireless communication systems. Therefore, mitigating such time varying fading effects is an essential condition for efficient data transmission and communication. However, CSI is not permitted at the receiver.

Thus, in practice accurate estimate of the CSI has a significant impact on the whole system working performance [1]. In contrast to the typical static or predictable characteristics of a wired communication channel, the wireless communication channel is dynamic or unpredictable, which provides an accurate analysis of the wireless communication system is always too strenuous. For a wireless communication system, between two antennas, Radio Frequency signal communication commonly suffers from the power loss, which affects its grade of performance. That power loss between them is a result of three different criteria: 1) Distance-dependent decrease of the power density called space attenuation OR path loss 2) And the absorption due to the molecules in the atmosphere conditions and 3) signal fading caused by terrain and weather conditions in the propagation of the signals. Section II discusses the OFDM basic model with cyclic prefix. Section III establishes the estimation procedure of the channel, based on block-type pilot arrangements. Section IV reviews the parameters with results of simulations. Section V sums up the result.

II. OFDM BASIC MODEL

Figure 1 OFDM basic block DIA. With cyclic prefix

The basic OFDM system blocks diagram with cyclic prefix. Under the assumption of frequency domain equalizations shown in Fig.1. The binary information is being generated from uniformly distributed random integers with equal probability of either 0 or 1 given as [5]:

\[ d_k = [d_0, d_1, d_2, ..., d_{N-1}] \]

\[ k = 0, ..., N - 1 \]
\[
x(k) = \sum_{n=0}^{N-1} S(n) \sin \left( \frac{2\pi kn}{N} \right) - j \sum_{n=0}^{N-1} S(n) \cos \left( \frac{2\pi kn}{N} \right)
\]

\[dk\] is converted from serial bit stream to parallel and mapped according to the modulation in the block of constellation mapper.

Where, \( S(n) \) is BPSK/QPSK symbols and \( N \) is the length of IDFT.

After the IFFT block, Cyclic Prefix of length assumed to be greater than the impulse response of the channel.

It is to be used to combat inter-symbol interference and inter-carrier interference (ICI) given as:

\[
\begin{bmatrix}
    x_{cp}(k) \\
    x(k)
\end{bmatrix}
\]

The OFDM signal is then constructed by applying the symbol along with CP to parallel to Serial converter. It is then transmitted on channel given as:

\[
y(k) = x(k) \otimes h(l) + n(k)
\]

Where \( h(l) \) is the channel impulse response. The length of channels should be shorter than the cyclic prefix. The noise generated by the OFDM system is:

\[
n(k) = \frac{-E_s}{N_0} * AWGN
\]

Where \( E_s \) is symbol to error ratio (SER) given as

\[
\left( \frac{E_s}{N_0} \right)_{dB} = \left( \frac{N}{N_{cp}+N} \right)_{dB} + \left( \frac{N_{int}}{N} \right)_{dB} + \left( \frac{E_b}{N_0} \right)_{dB}
\]

Here, \( N_{cp} \) represents the length of cyclic prefix, \( N_{int} \) is the No. Of use subcarriers and \( N \) is the length of FFT or No. Of subcarriers [5]. Since the OFDM signal has overhead in terms of CP, so to compensate for it, we have to scale it so that resultant OFDM signal received

\[
r(k) = \sqrt{\frac{N_{cp}+N}{N}} x y(k)
\]

is:

At the receiver the reverse steps are involved and since the OFDM symbols were circularly convolved with channel IR, so after FFT at the receiver [6], the received data are analyzed by using the frequency domain equalizer and the equation given as:

\[
\hat{x}(k) = \frac{y(k)}{h(k)}
\]

Where \( H(k) \) is a response of the channel in frequency-domain. The frequency domain equalization is applied to the symbols faded as a result of experiencing multipath. Section V [7] dealt with the results.

III. CHANNEL ESTIMATION PROCEDURE

Figure 2: Channel Estimation using LS/MMSE algorithm

Figure 2 illustrates the OFDM system for block-type channel estimation. In block-type pilot based channel estimation, each subcarrier in an OFDM symbol is employed in such a way that all subcarriers are used as pilots. The estimation of the channel is performed using Least Square estimator and Minimum Mean Square Error estimator [8], [9]. The system in Fig.2 is modeled using the following equation:
The vector \( h / \sqrt{N} \) is the observed channel impulse response when the frequency response of \( h(t) \)

\[
\hat{h}/ \sqrt{N} = \frac{1}{\sqrt{N}} \sum_{m} e^{-j \theta_m} \frac{\sin(\pi N m)}{\sin(\pi m)} \]

Where, \( m \) is the length of taps. \( N \) is the No. of subcarriers.

And \( \theta_m \) is the value of the tap. If intersymbol interference is eliminated by the cyclic prefix, then the system in the Fig. 2 can be modeled using the equation given as [6]

\[
y_k = H_k x_k + w_k, \quad k = 0, \ldots, N-1
\]

Where \( H_k \) is the Frequency response of \( h \) given by:

\[
H = [H_0 \quad H_1 \quad \ldots \quad H_{N-1}]^T = DFT_N(h)
\]

\[
w = [w_0 \quad w_1 \quad \ldots \quad w_{N-1}]^T = DFT_N(w)
\]

Now writing the (11) in Matrix form, it becomes:

\[
y = XFh + w
\]

Here,

\[
X = \text{diag} \{ x_0 \quad x_1 \quad \ldots \quad x_{N-1} \}
\]

\[
y = [y_0 \quad y_1 \quad \ldots \quad y_{N-1}]^T
\]

\[
w = [w_0 \quad w_1 \quad \ldots \quad w_{N-1}]^T
\]

\[
h = [h_0 \quad h_1 \quad \ldots \quad h_{N-1}]^T
\]

\[
F = \begin{bmatrix}
W_0^N & \cdots & W_{N-1}^N \\
W_0^{2N} & \cdots & W_{N-1}^{2N} \\
\vdots & \ddots & \vdots \\
W_0^{(N-1)N} & \cdots & W_{N-1}^{(N-1)N}
\end{bmatrix}
\]

\[
F^{*} = \begin{bmatrix}
W_0^N & \cdots & W_{N-1}^N \\
W_0^{2N} & \cdots & W_{N-1}^{2N} \\
\vdots & \ddots & \vdots \\
W_0^{(N-1)N} & \cdots & W_{N-1}^{(N-1)N}
\end{bmatrix}
\]

If the channel vector \( h \) is Gaussian and is not correlated with the noise of the channel \( W \), then the frequency domain MMSE estimates of \( h \) becomes [5].

\[
R_{\text{MMSE}} = E[F_{\text{H}0}^T y]
\]

Where,

\[
R_{\text{H}} = E[h^H h] = R_{\text{H}}^T R_{\text{H}}^T
\]

\[
R_{yy} = E[y y^H] = X F R_{\text{H}} h y^H + \sigma_n^2 I_N
\]

Here, \( R_{\text{H}} \) is the cross correlation matrix between \( h \) and \( y \).

\( R_{\text{H}} \) is the auto correlation matrix of \( h \) with itself and \( \sigma_n^2 \) is the variance of \( \epsilon \). The factors \( R_{\text{H}} \) and \( \sigma_n^2 \) are considered to be known.

The LS estimate of the channel is given as:

\[
\hat{h}_{\text{LS}} = (X^H X)^{-1} X^H y
\]

Which minimizes \( (y - XFh)^H (y - XFh) \).

Both estimators suffer from different drawbacks. The MMSE usually suffers from a high complexity, where the LS estimator suffers from mean-square- error which is high. The MMSE estimator is required to calculate an \( N \times N \) matrix which results in a high complexity when becomes large [5].

Note that both Estimators are derived under the assumption of known channel correlation and noise variance. In actual fact, these quantities \( R_{\text{H}} \) and \( \sigma_n^2 \) are either regarded as fixed or estimated most commonly.

\[\text{IV SIMULATIONS AND RESULTS}\]

This section discusses the findings from simulation performed based on the information and mathematics in Section II & III respectively. For the simulations of a basic OFDM system, we used the following parameters as shown in Table 1.
The Fig. 3 and Fig. 4 shows the comparison of the BER (Bit-Error-Rate) with different SNR’s on BPSK and QPSK constellation using 3 different channel models described in the Table.

Figure 3: Comparison of the BER for BPSK in AWGN/FNS/Rayleigh channel
The above figure shows comparison of the BER at the three different channels. For small SNR values the calculated BER is quite substantial due to the high power of noise. As SNR increases, the BER decreases as shown. Similarly for QPSK, again the BER determines how many of the received bits are in error, and then computes it by the number of bits in error divided by the total No. Of bits in the transmitted signal.

Figure 4: Comparison of the BER for QPSK in AWGN/FNS/Rayleigh channel
The Fig. 4 shows the comparison of BER on three different channels. For small SNR values the calculated BER is quite large due to the higher power of noise. As the SNR increases, the BER decreases. As the BER for the Multipath fading is simulated for the (No. Of taps) = 8, which is less than the length of the CP, however, if we increase the number of taps for the multipath fading then the resultant BER curve would show that the performance is getting worse and more errors would occur.

Figure 5: Comparison of the BER of QPSK for different no. Of taps
Fig. 5 shows this effect of performance degradation by increasing the number of taps in the multipath channel. The result shows that as we increase the No. Of taps, transmitted signal that undergoes high degradation.
The Fig. 6 shows the MSE versus SNR for the LS and MMSE Estimators. For low SNR’s channel noise effect is higher than the approximation effect, while it becomes dominant for large SNR’s. Because of the No. Of time it would be reflected by the multipath (No. Of taps).

Bit-Error-Rate curves are based on the mean square errors of the channel estimation.

For the calculation of BER, the simulation makes the most of the formulae calculated earlier. In the simulation, we first transmitted the training symbols just to estimate the behavior of the channel so that these results can be utilized again for the actual transmission in the simulation code. Fig. 7 shows the BER of the OFDM system using LS and MMSE estimation for 3-taps.

The Channel estimation is one of the Fundamental issues of OFDM system design. The transmitter signal undergoes numerous effects such reflection, refraction and diffraction. Also, due to the mobility, the channel response can change rapidly over time. At the receiver these channel effects must be canceled to recover the original signal. The MMSE performs better than the LS utilizing 3-taps where the performance metric is mean- square and symbol error rate.

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