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A TWO UN1T REDUNTANT SYSTEM WITH TWO WAY REPAIR FACILITY

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***Abstract*— A two unit redundant system is studied, in which one unit is operative and the other is at standby which replace the failure unit instantaneously. To increase system availability, the failure rate of the operative unit and the repair rate of the failed unit adjust automatically according to the standby unit. A two stage repair facility is available for the extent of failure, the repair facility by regular repairman and repair facility by expert repairman. The repair facility is prior with sending first failed unit to regular repairman and if damage is so serious then unit will be sent to expert repairman. Still the damage is not recovered then unit can be replaced with warm standby unit. Waiting facility is also available failure and replacement also. Using regenerative point technique in the markov renewal process, transition probabilities, mean sojourn time and mean time to system failure are obtained.**

***Keywords*— Cloud computing, Multi-tenancy, Virtualization, Cloud resource monitoring, simulation.**

I. INTRODUCTION

This paper studied the system which is time and money saver and is very close to real and practical aspects used by the system designers. In order to save money and time of the system a regular repairman facility is available in the system round the clock. When system failure cannot be overcome by regular repairman expert repairman is called upon. If the system is in completely down state mode repairing facilities rates can be improved so as to make down state in to operative state. With the help of regenerative point technique various characteristics of interest are obtained.

2. MODEL DESCRIPTION AND ASSUMPTIONS

1 The system consists of two identical units. Initially one unit is operative and the other is a warm standby.
2 Upon failure of an operative, the warm standby unit becomes operative instantaneously.
3 Single way repair facility is available in which firstly failed unit goes to repair by regular repairman and if difficulty cannot be overcome by regular repairman it is sent to expert repairman.
4 While going for repairing and repair facilities are busy then •the failed unit will wait for its chance.
5 The repairing rate of the failed unit increases at the down state o the system.
6 Failure rates of operative and down and warm stand by unit-are constant. The rates of repair and replacement are constant in both up and down state of the system.

3. NOTATIONS AND STATES

α Failure rate when stand by is available.
$\alpha 1$ failure rate when stand by is not available.
θ Failure rate of warm stand by unit.
β Repairing rate of regular repairman.
$\beta 1$ waiting rate for regular repairman.
γ Repairing rate of expert repairman.
γ 1 waiting rate for expert repairman.
δ Replacement rate.
$\delta 1$ waiting rate for replacement.
W Normal unit is operative
Wo Normal unit kept as warm stand by
Frr failed unit under regular repairman
Fwrr failed unit waiting for regular repairman
Fer failed unit under expert repairman
Ferp failed unit under replacement
Fwrep failed unit waiting for expert repairman
Fwer failed unit waiting for expert repairman

4. POSSIBLE TRANSITIONS

Up states

$S_0(W_o, W_{cs})$;$S_1(F_{rr}, W_o)$;$S_2(W_o, F_{wrr})$; $S_6(Fer, Wo)$;$S_9(W_o, F_{wrep})$;
$S_5(F_{rep}, W_o)$

Downstates

$S_4\ (F_{rr}, F_{wrr})$;$S_7\ (F_{wer}, F_{wrr})$; $S_8(F_{er}, F_{wrr})$; $S_3(F_{er}, F_{rr})$; $S_{10}\ (F_{rep}, F_{rr})$;

5. TRANSITION PROBABILITIES

The epochs of entry into states So, S1, S2, S5, S6, S9 are regenerative points and E is the set of the states. let To (=0), T1, T2 denotes the entry into the states si e E. let Xn be the states visited at epochs i.e. just after the transition at Tn. then [Xn, Tr].Markov renewal process with state space E and

$$Q_{ij}(t) = \Pr[X_{n+1}=S_j, T_{n+1}-T_n \leq t \mid X_n = S_i]$$

Is the semi-markov kernel over E. the stochastic matrix of the embedded Markov chain is P = (P ii) = Q (09)=Q (co) ...(2)

The non zero transition probabilities of transition are calculated below with specific
rates assumed above.

$$P_{16}=\frac{\alpha_1}{\beta+\alpha_1} \quad P_{14}=\frac{p\beta}{\beta+\alpha_1} \quad P_{10}=\frac{q\beta}{\beta+\alpha_1}$$

$$P_{01}=\frac{p\alpha}{\alpha+\beta_1} \quad P_{02}=\frac{\beta_1}{\alpha+\beta_1} \quad P_{04}=\frac{q\alpha}{\alpha+\beta_1} \quad P_{21}=1$$

$$P_{31}=\frac{\gamma}{\gamma+\partial} \quad P_{35}=\frac{\partial}{\gamma+\partial}$$

$$P_{43}=\frac{q\beta_1}{\gamma+\beta_1} \quad P_{42}=\frac{q\gamma_1}{\gamma+\beta_1} \quad P_{47}=\frac{p\gamma}{\gamma+\beta_1}$$

$$P_{48}=\frac{p\beta_1}{\gamma+\beta_1} \quad P_{5\,10}=\frac{\alpha_1}{\delta+\alpha_1} \quad P_{95}=\frac{\delta}{\delta+\alpha_l}$$

$$P_{65}=\frac{\alpha_1}{\alpha_1+\gamma_1} \quad P_{69}=\frac{\gamma_1}{\alpha_1+\gamma_1}$$

$$P_{78}=1 \quad P_{83}=1 \quad P_{9\,10}=1 \quad P_{10,1}=1$$

The above transition probability easily suggest that
P16+ P1+ P10 = 1, P01+ P02+ Po4=1,
P21=1 P31 + P35 =1, P43+ P42+P47 P48 =1,
P5 10 + P59=1, P65 + P69 =1, P78=1, P83-1, P9 10-1,P10,1— 1

6. MEAN SOJOURN TIME
The Mean sojourn time in a state Si is defined as the length of stay in time in a state Si, before transiting to any other state

S_i, before transiting to any other state

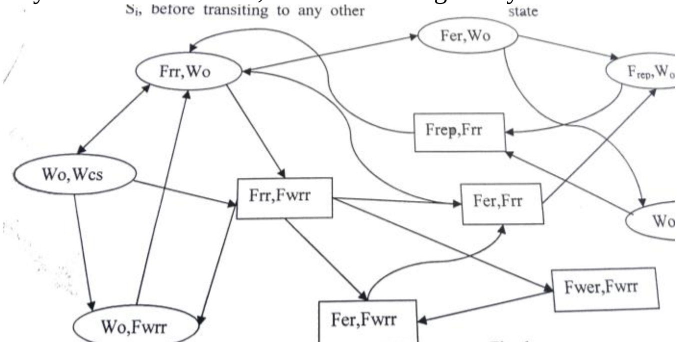

Fig. 1

If T denotes the soujourn time in S_i then

If T denotes the soujourn time in S_i, then

$$\mu_i = E(T) = \int_0^\infty \Pr[T > t]dt \text{ in state}$$

S_i = (0,1,2,3………..10) are

$$\mu_0=\frac{1}{\theta+\alpha} \quad \mu_1=\frac{1}{\beta+\alpha_1} \quad \mu_2=\frac{1}{\alpha_1+\beta_1}$$

$$\mu_3=\frac{1}{\beta+\beta_1} \quad \mu_4=\frac{1}{\nu+\alpha_1} \quad \mu_5=\frac{1}{\delta+\alpha_1}$$

$$\mu_6=\frac{1}{\nu_1} \quad \mu_7=\frac{1}{\nu_1+\beta_1} \quad \mu_8=\frac{1}{\delta_1+\alpha_1}$$

$$\mu_9=\frac{1}{\gamma+\beta} \quad \mu_{10}=\frac{1}{\delta+\beta}$$

7. MTSF(mean time to system failure)

To investigate the distribution function 7T, (t) of the time to system failure with starting state Si, we regard the failed state as absorbing. On the basis of arguments used for regenerative process, we obtain the following relations for 7, (t).

$$\pi_0(t)= Q_{01}(t) \$ \pi_1(t) + Q_{02}(t) \$ \pi_2(t) + Q_{04}(t)$$

$$\pi_1(t)= Q_{10}(t) \$ \pi_0(t)+ Q_{14}(t) + Q_{16}(t) \$ \pi_6(t)$$

$$\pi_2(t)= Q_{21}(t) \$ \pi_1(t)$$

$$\pi_5(t)= Q_{5\,10}(t)$$

$$\pi_6(t)= Q_{69}(t) \$ \pi_9(t) + Q_{65}(t) \$ \pi_5(t)$$

$$\pi_9(t)= Q_{95}(t) \$ \pi_5(t) + Q_{9\,10}(t)....(36\text{-}41)$$

On taking laplace — stieljielts transform of (36-41) relations and solving for izo (s), we have

$$\tilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)}$$

$$N_1(s) = \tilde{Q}_{01}(s)\,\tilde{Q}_{14}(s) + \tilde{Q}_{01}(s)\,\tilde{Q}_{16}(s)\,\tilde{Q}_{65}(s)\,\tilde{Q}_{510}(s) + \tilde{Q}_{02}(s)\,\tilde{Q}_{21}(s)\,\tilde{Q}_{14}(s) + \tilde{Q}$$
$$\tilde{Q}_{16}(s)\,\tilde{Q}_{69}(s)\,\tilde{Q}_{910}(s) + \tilde{Q}_{01}(s)\,\tilde{Q}_{16}(s)\,\tilde{Q}_{69}(s)\,\tilde{Q}_{95}(s)\,\tilde{Q}_{510}(s) + \tilde{Q}_{02}(s)\,\tilde{Q}_{21}($$
$$\tilde{Q}_{69}(s)\,\tilde{Q}_{95}(s)\,\tilde{Q}_{510}(s) + \tilde{Q}_{02}(s)\,\tilde{Q}_{21}(s)\,\tilde{Q}_{16}(s)\,\tilde{Q}_{69}(s)\,\tilde{Q}_{910}(s)$$
...(43)

$$D_1(s) = 1-[\tilde{Q}_{01}(s)\,\tilde{Q}_{14}(s)\,\tilde{Q}_{42}(s)\,\tilde{Q}_{21}(s)\,\tilde{Q}_{10}(s) + \tilde{Q}_{02}(s)\,\tilde{Q}_{21}(s)\,\tilde{Q}_{10}(s)]$$

$$MTSF = E(T) = -\frac{d\,\tilde{\pi}_0(s)}{ds}\Big|_{s\to 0}$$
$$= -\frac{D_1'(0) - N_1'(0)}{D_1(0)} = \frac{N_1}{D_1},$$

Where,

$$N_1 = m_0 + P_{02}P_{21}m_1 + P_{01}m_1 + P_{01}P_{16}m_5 + P_{01}P_{16}P_{65}m_5 + P_{01}P_{16}P_{69}m_9 + P_{02}P_{21}P_{16}P_{69}m_9 \quad \ldots(46)$$

$$D_1 = 1 - [P_{01}P_{14}P_{42}P_{21}P_{10} + P_{02}P_{21}P_{10}] \quad \ldots(47)$$

8. SYSTEM AVAILBILITY

As defined earlier, A, (t) is the probability that the system having started from a regenerative state Si at t =0, is now under repair. By probability argument we have

$$A_0(t) = M_0(t) + q_{01} © A_1(t) + q_{02}(t) © A_2(t) + q_{04}(t) © A_4(t)$$
$$A_1(t) = M_1(t) + q_{14}(t) © A_4(t) + q_{16}(t) © A_6(t)$$
$$A_2(t) = M_2(t) + q_{21}(t) © A_1(t)$$
$$A_3(t) = q_{35}(t) © A_5(t) + q_{31}(t) © A_1(t)$$
$$A_4(t) = q_{48}(t) © A_8(t) + q_{43}(t) © A_8(t) + q_{47}(t) © A_7(t) + q_{42}(t) © A_2(t)$$
$$A_5(t) = M_5(t) + q_{5\,10}(t) © A_{10}(t)$$
$$A_6(t) = M_6(t) + q_{65}(t) © B_5(t) + q_{69}(t) © A_9(t)$$
$$A_7(t) = q_{78}(t) © A_8(t)$$
$$A_8(t) = q_{83}(t) © A_3(t)$$
$$A_9(t) = M_9(t) + q_{9\,10}(t) © A_{10}(t)$$
$$A_{10}(t) = q_{10\,1}(t) © A_1(t)$$

Where ,

$$M_0(t) = \exp[-(\theta+\alpha)t]$$
$$M_1(t) = \exp[-(\beta+\alpha)t]$$
$$M_2(t) = \exp[-(\beta_1+\alpha_1)t]$$
$$M_6(t) = \exp[-(\alpha_1+\gamma)t]$$
$$M_5(t) = \exp[-(\alpha_1+\delta)t]$$
$$M_9(t) = \exp[-(\delta_1+\alpha_1)t]$$

Taking laplace transforms of equations (48-58) equations and solving them for

Ao*(s) =N2(s)/D2(s) …………..(59)

$$A_0^*(s) = N_2(s)/D_2(s) \quad \ldots(59)$$

$$N_2(s) = M_0^*(t) - M_0^*(t)q_{16}^*q_{69}^*q^*_{9\,10} - M_0^*(t)q_{16}^*q_{65}^*q_{5\,10}^*q_{10\,1}^* + M_0^*(t)q_{02}^*q_{21}^*q_{10}^* - M_0^*(t)q_{04}^*q_{42}^*q_{21}^*q_{10}^* + M_1^*(t)q_{10}^*q_{02}^*q_{21}^* + M_0^*(t)q_{01}^*q_{16}^*q_{69}^*q_{95}^* + M_1^*(t)q_{16}^*q_{69}^*q_{95}^* \quad \ldots(60)$$

$$D_2(s) = 1 - q^*_{10}q^*_{02}q^*_{21} + q^*_{04}q^*_{42}q^*_{21}q^*_{10} + q^*_{01}q^*_{16}q^*_{65}q^*_{5\,10}q^*_{10\,1}q^*_{10} + q^*_{16}q^*_{65}q^*_{5\,10}q^*_{10\,1} + q^*_{01}q^*_{16}q^*_{69}q^*_{9\,10}q^*_{10\,1}q^*_{10} + q^*_{01}q^*_{14}q^*_{43}q^*_{31}q^*_{10} + q^*_{14}q^*_{43}q^*_{31} + q^*_{04}q^*_{48}q^*_{83}q^*_{31}q^*_{10} + q^*_{14}q^*_{43}q^*_{31} + q^*_{01}q^*_{14}q^*_{48}q^*_{83}q^*_{31}q^*_{10} + q^*_{01}q^*_{14}q^*_{48}q^*_{83}q^*_{35}q^*_{5\,10}q^*_{10\,1}q^*_{10} + q^*_{04}q^*_{47}q^*_{78}q^*_{83}q^*_{31}q^*_{10} + q^*_{01}q^*_{14}q^*_{47}q^*_{78}q^*_{83}q^*_{31} + q^*_{01}q^*_{14}q^*_{47}q^*_{78}q^*_{83}q^*_{35}q^*_{5\,10}q^*_{10\,1}q^*_{10} - q^*_{01}q^*_{16}q^*_{69}q^*_{95} - q^*_{02}q^*_{21}q^*_{16}q^*_{69}q^*_{95} + q^*_{01}q^*_{14}q^*_{42}q^*_{21} + q^*_{02}q^*_{21}q^*_{14}q^*_{42}q^*_{21}q^*_{10}. \quad \ldots(61)$$

The steady state availability when the system starts operation from Si is thus as follows

$$A_0(\infty) = \lim_{s\to 0} s.\,A_0^*(s) = N_2/D_2 \quad \ldots(62)$$

$$N_2(s) = [1 - p_{16}p_{69}p_{9\,10}p_{10\,1} - p_{16}p_{65}p_{95\,10}p_{10\,1} + p_{02}p_{21}p_{10} - p_{04}p_{42}p_{21}p_{10} + p_{01}p_{16}p_{69}p_{95}]\mu_0 + [p_{10}p_{02}p_{21} + p_{16}p_{69}p_{95}]\mu_1 \quad \ldots(63)$$

$$D_2(s) = m_0p_{14} + m_0p_{21}p_{14} + m_0p_{16}p_{65}p_{5\,10} + m_0p_{04} - m_1p_{02}p_{21}p_{10} - m_0p_{16}p_{65}p_{5\,10}p_{10\,1} - m_0p_{14}p_{47}p_{78}p_{83}p_{31} - m_0p_{14}p_{43}p_{35}p_{5\,10}p_{10\,1} + m_1p_{48}p_{83}p_{31} - m_2p_{10}p_{02}p_{21} + m_2p_{04}p_{42} + m_4p_{83}p_{31}p_{14} + m_4p_{47}p_{78}p_{83} + m_3p_{5\,10}p_{10\,1}p_{14} - m_5p_{61}p_{14}p_{43}p_{35} + m_6p_{5\,10}p_{10\,1}p_{16} - m_7p_{83}p_{35}p_{5\,10}p_{10\,1}p_{14}p_{47} - m_8p_{35}p_{5\,10}p_{10\,1}p_{14}p_{48}p_{83} + m_8p_{31}p_{14}p_{48} - m_9p_{5\,10}p_{10\,1}p_{16}p_{65} - m_9p_{10\,1}p_{16}p_{65}p_{5\,10} - m_9p_{10\,1}p_{14}p_{43}p_{35}p_{5\,10} - m_9p_{10\,1}p_{14}p_{48}p_{83}p_{35}p_{5\,10} - m_9p_{10\,1}p_{14}p_{47}p_{78}p_{83}p_{35}p_{5\,10} - m_{10}p_{16}p_{65}p_{5\,10} \quad \ldots(64)$$

BUSY PERIOD ANALYSIS:

As defined earlier, Bi (t) is the probability that the system having started from a regenerative state Si at t ----O, is now under repair. By probability argument we have

$$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t) + q_{04}(t) © B_4(t)$$
$$B_1(t) = w_1(t) + q_{14}(t) © B_4(t) + q_{13}(t) © B_3(t) + q_{01}(t) © B_1(t) + q_{10}(t) © B_0(t) + q_{16}(t) © B_6(t)$$
$$B_2(t) = w_2(t) + q_{21}(t) © B_1(t)$$
$$B_3(t) = w_3(t) + q_{32}(t) © B_2(t) + q_{39}(t) © B_9(t) + q_{36}(t) © B_6(t) + q_{37}(t) © B_7(t)$$
$$B_4(t) = w_4(t) + q_{45}(t) © B_5(t) + q_{48}(t) © B_8(t) + q_{42}(t) © B_2(t) + q_{48}(t) © B_8(t) + q_{47}(t) © B_7(t) + q_{43}(t) © B_3(t)$$
$$B_5(t) = w_5(t) + q_{50}(t) © B_0(t) + q_{5,10}(t) © B_{10}(t)$$
$$B_6(t) = w_6(t) + q_{67}(t) © B_7(t) + q_{69}(t) © B_9(t)$$
$$B_7(t) = w_7(t) + q_{79}(t) © B_9(t)$$
$$B_8(t) = w_8(t) + q_{85}(t) © B_3(t)$$
$$B_9(t) = w_9(t) + q_{91}(t) © B_1(t) + q_{95}(t) © B_5(t) \quad \ldots(65\text{-}75)$$
$$B_{10}(t) = w_{10}(t) + q_{10\,1}(t) © B_1(t)$$

$$W_1(t) = \exp[-(\beta+\alpha_1)]$$
$$W_2(t) = \exp[-(\beta_1+\alpha_1)]$$
$$W_3(t) = \exp[-(\beta_1+\beta)]$$
$$W_4(t) = \exp[-(\alpha_1+\gamma)]$$
$$W_5(t) = \exp[-(\alpha_1+\delta)]$$
$$W_6(t) = \exp[-(\gamma_1+\beta_1)]$$
$$W_7(t) = \exp[-(\gamma+\beta_1)]$$
$$W_8(t) = \exp[-(\delta_1+\alpha_1)]$$
$$W_9(t) = \exp[-(\beta+\gamma)]$$
$$W_{10}(t) = \exp[-(\beta_1+\delta)]$$

Taking laplace transforms of equations (65-75) relations and solving for Bo*(s)

Bo = lim Bo (t) = lim s. Bo*(s) = N3 / D2, s—>co .....(85)

Where

D2 is same as in availability analysis

$$\begin{aligned}
N_3 = \; & p_{01} w_1^* + p_{01}[\{p_{16} p_{65} p_{59} p_{910} p_{101} + p_{14} p_{48}^{(7)} (p_{83} p_{31} + p_{83} p_{35} p_{510} p_{101})\} + p_{04} p_{83}\{p_{48} p_{31} + p_{48}^{(7)} p_{31}\}] w_1^* + p_{01} p_{14} (p_{48} p_{83} + p_{47} p_{78} p_{83}) w_3^* + \\
& p_{10} p_{02} p_{21} p_{14} p_{43} w_3^* + p_{01} q_{14}^* w_4^* + p_{10} p_{02} p_{21} p_{14} w_4^* - p_{04}[p_{01} p_{14} p_{42} \\
& p_{21} p_{10} - p_{14} p_{43} p_{31} - p_{01} p_{16} (p_{65} p_{59} p_{910} p_{101} + p_{69} p_{910} p_{101})\} - p_{14} p_{02} p_{21} \\
& p_{16} p_{69} p_{910} p_{101} (p_{10} + 1) - p_{01} p_{14} \{p_{21} p_{16} p_{69} p_{910} p_{101} p_{10} p_{02} + p_{65} p_{59} \\
& p_{910} p_{101} (p_{10} + 1) - p_{65} p_{510} p_{101} (p_{10} p_{02} + 1) - p_{69} p_{910} p_{101}\}] w_4^* + p_{01} p_{16} w_6^* \\
& + p_{01}(p_{16}^* p_{65} + p_{01} p_{14} p_{47} p_{78} p_{83} p_{35}) w_5^* + (q_{01}^* q_{14}^* q_{48}^* + p_{10} p_{02} p_{21} \\
& p_{14} p_{48}) w_8^* + p_{01} (p_{16} p_{65} p_{59} + p_{14} p_{47} p_{78} p_{83} p_{35} p_{59}) w_9^* + p_{04} p_{47} p_{78} p_{83} \\
& p_{35} p_{59} w_9^* + [p_{01} p_{16} p_{65} p_{59} p_{910} + p_{01} p_{14} p_{47} p_{78} p_{83} p_{35} p_{510} + p_{10} p_{02} p_{21} \\
& p_{16} p_{69} p_{910} + p_{04} p_{47} p_{78} p_{83} p_{35} p_{59} p_{910} - p_{01} p_{16} p_{65} p_{510} p_{14}(p_{43} p_{31} + p_{48} \\
& p_{83} p_{31}) + p_{01}\{p_{16} p_{65}(p_{59} p_{910} - p_{510} p_{14} p_{47} p_{78} p_{83} p_{83} p_{31}) + p_{14} p_{47} p_{78} p_{83} \\
& p_{35} p_{510}\}] w_{10}^* \qquad \ldots(86)
\end{aligned}$$

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