

TWO UNIT REDUNDANT SYSTEM WITH n -FAILURE MODES, n - FAULT DETECTION AND INSPECTION

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Abstract— The present paper gives the analysis of a two unit cold standby system with n failure modes of the online unit. The failed online unit is first sent for fault detection to decide its failure mode. After repair the unit is sent for inspection to decide whether the repaired unit is perfect or not. If it is found to be imperfect, them it is sent for post repair. Using regenerative points technique in Markov renewal process several effective measures of reliability are obtained.

Keywords— Laws - legislations - quality of life - sustainability.

I. INTERODUCTION

Many researchers working in the field of system reliability have analyzed two unit standby redudiciant system without considering the mode of failure of an online unit (operative unit). But in our daily life, we come across with engineering systems in which the online unit has more than one failure modes. Incorporating the idea of failure mode, we in,, this paper analysed a two unit system with n -failure modes of the on line unit. The failed unit is repaired, in the respective mode and after each repair it is inspected to decide whether the repaired unit is perfect or not If the repaired unit is found to be imperfect than it is sent. for post repair.

II. MODEL DESCRIPTION AND ASSUMPTIONS

- (1) Consider a two unit system with one unit operative and the other as cold standby.
- (2) Upon failure of an operative unit, the cold standby unit becomes operative instantaneously
- (3) Before starting the repair, a failed unit is taken for its fault detection t decide the mode of failure. (4) After repair, the unit is inspected to decide whether the repair is perfect or not. If the repair of a unit is found to be perfect then the repaired unit becomes operative or cold standby otherwise it is again sent for post repair. The probability of having perfect repair is fixed.
- (5) Failure rate of an operative unit is constant, while the distributions of time for fault detection, repair, inspection and post-repair are general .
- (6) A single repair facility is available for fault detection, repair, inspection and post-repair.
- (7) Service discipline is FCFS.

Identifying the suitable regenerative points several measures of system effectiveness are discussed.

III. NOTATIONS AND STATES OF THE SYSTEM

N_0 : Normal unit when it is operative

N_s : Normal unit when it is cold standby

F_{wf} : Failed unit waiting for fault detection

F_f : Failed unit under fault detection

FF : Fault detection is continued from earlier state

F_{rc} : Failed unit in the C_{th} failure mode is under repair

F_{ic} : Failed unit in the C_{th} failure mode is under inspection

F_{pc} : Failed unit in the C_{th} failure mode is under post repair

Note : C takes value from 1 to n .

α : Constant failure rate of an operative unit.

$F(\cdot), F_c(\cdot)$: pdf and cdf of time to detect the mode of failure

$G_c(\cdot), G_c(\cdot)$: pdf and cdf of time for repair in the C_{th} failure mode.

$K_c(\cdot), K_c(\cdot)$: pdf and cdf of time for inspection in the C_{th} failure mode.

$H_c(\cdot), H_c(\cdot)$: pdf and cdf of time for post repair in the C_{th} failure mode

Two Unit Redudent System With n -Failure Modes

$P = (1-q)$: Probability that the repair is perfect.

$m_1, m_{2c}, m_{3c}, m_{4c}$: Mean time for fault detection, repair inspection and post repair in the C_{th} failure mode.

Using the above notations, the possible states of the system are :

Up States :

S_0 : (N_0, N_s) ; S_1 : (N_0, Fr) ; S_{3c} (N_0, Frc)

S_{5c} : (N_0, Fic) ; S_{7c} : (N_0, Fpc)

Down States:

$S_e : (F_{wf}, FF) ; S_{4c} : (F_{wf}, F_{rc}) ; S_{6e} : (F_{wf}, F_{ie})$

$S_{ec} : (F_{wf}, F_{pc})$

The possible transitions between the states are shown in Fig-1.

IV. TRANSITION AND STEADY STATE PROBABILITIES

Using the definition of $Q_{ij}(t)$ the transition probability matrix of the embedded Markov chain is

$$P = (P_{ij} = [Q_{ii}(\infty)]_i)$$

with the non zero elements :

with the non zero elements :

$$\begin{aligned}
 P_{01} &= 1 & ; & & P_{12} &= 1 - f^*(\alpha) \\
 P_{1,3c} &= f^*(\alpha) & ; & & P_{3c,4c} &= 1 - g^*_{c0}(\alpha) \\
 P_{3c,5c} &= g^*_c(\alpha) & ; & & P_{4c,6c} &= 1 \\
 P_{5c,0} &= PK^*_c(\alpha) & ; & & P_{5c,6c} &= 1 - K^*_c(\alpha) \\
 P_{5c,7c} &= qK^*_c(\alpha) & ; & & P_{6c,1} &= P \\
 P_{6c,8c} &= q & ; & & P_{7c,0} &= h^*_c(\alpha) \\
 P_{7c,8c} &= 1 - h^*_c(\alpha) & ; & & P_{8c,1} &= 1 \\
 P_{1,4c} &= 1 - f^*(\alpha) & ; & & P_{3c,6c} &= 1 - g^*_c(\alpha)
 \end{aligned}$$

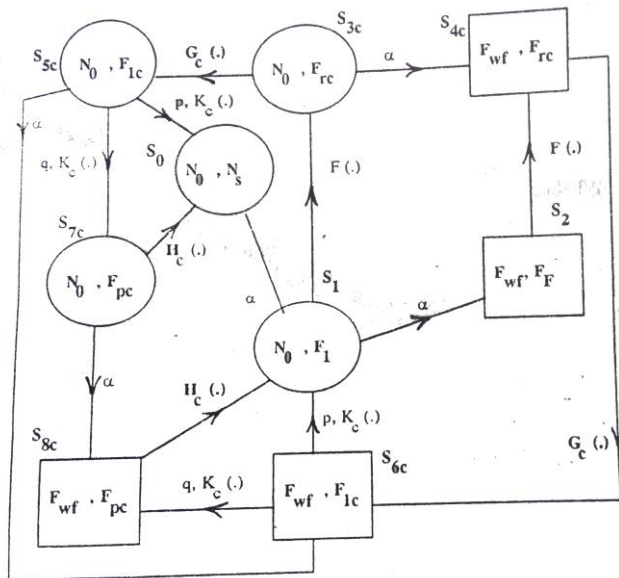


Fig - 1

○ : UP STATE
□ : DOWN STATE

$$\begin{aligned}
 P_{5e,1} &= P[1 - K^*_c(\alpha)] & ; & & P_{5c,8c} &= q[1 - K^*_c(\alpha)] \\
 P_{7c,1} &= 1 - h^*_c(\alpha) & & & & \dots (2-20)
 \end{aligned}$$

V. MAN SOJOURN TIMES

As defined earlier, the mean sojourn time in state $S_1 \in E$ is

$$\mu_1 = E(T) = \int P[T > t] dt$$

Thus we have

$$\begin{aligned}
 \mu_0 &= 1/\alpha & ; & & \mu_1 &= [1 - f^*(\alpha)]/\alpha \\
 \mu_{3c} &= [1 - g^*_c(\alpha)]/\alpha & ; & & \mu_{4c} &= \int t g_c(t) dt \\
 \mu_{5c} &= [1 - k^*_c(\alpha)]/\alpha & ; & & \mu_{6c} &= \int t k_c(t) dt \\
 \mu_{7c} &= [1 - h^*_c(\alpha)]/\alpha & ; & & \mu_{8c} &= \int t h_c(t) dt \\
 & & & & & \dots (2-20)
 \end{aligned}$$

VI. MEAN TIME TO SYSTEM FAILURE

To obtain the distribution function $\Pi_1(t)$ of TSF with starting state S_i , we regard the failed states S_2, S_{4c}, S_{6c} and S_{8c} as absorbing.

Using the probabilistic arguments, the recursive relations among $\Pi_1(t)$

Are:

$$\begin{aligned}
 \Pi_0(t) &= Q_{01}(t) \$ \Pi_1(t) \\
 \Pi_1(t) &= Q_{12}(t) \$ \sum Q_{1,3c}(t) \$ \Pi_{3c}(t) \\
 \Pi_{3c}(t) &= Q_{3c,4c}(t) + \sum Q_{3c,5c}(t) \$ \Pi_{5c}(t) \\
 \Pi_{5c}(t) &= Q_{5c,6c}(t) + Q_{5c,0}(t) \$ \Pi_0(t) + \sum Q_{5c,7c}(t) \$ \Pi_{7c}(t) \\
 \Pi_{7c}(t) &= Q_{7c,8c}(t) + Q_{7c,0}(t) \$ \Pi_0(t) \\
 & \dots (30-34) \\
 \{ \sum \text{ extends from } c = 1 \text{ to } c = n \}
 \end{aligned}$$

Taking Laplace-Stieltjes transform of these relations and solving for Π_0 by omitting the argument "S" for brevity, therefore we get MTSF of the system when it starts operation from S_0 . So, as

$$E(T) = \left. \frac{-d \tilde{\Pi}_0(s)}{ds} \right|_{s=0} = \frac{D'_1(0) - N'_1(0)}{D_1(0)} = N_1/D_1 \dots (35)$$

where

$$N_1 = \mu_0 + \mu_1 + \sum P_{1,3c} (\mu_{3c} + P_{3c,5c} (\mu_{5c} + \mu_{7c} P_{5c,7c})) \dots (36)$$

and

$$D_1 = 1 - \sum P_{1,3c} P_{3c,5c} (P_{5c,0} + P_{5c,7c} P_{7c,0}) \dots (37)$$

VII. AVAILABILITY ANALYSIS

As define& the expression for $M_i(t)$ are

$$\begin{aligned} M_0(t) &= e^{-(\alpha + \beta)t} & ; & \quad M_1(t) = e^{-\alpha t} \bar{F}(t) \\ M_{3c}(t) &= e^{-\alpha t} \bar{G}_c(t) & ; & \quad M_{5c}(t) = e^{-\alpha t} \bar{K}_c(t) \\ M_{7c}(t) &= e^{-\alpha t} \bar{H}_c(t) & \dots & \quad (41-45) \end{aligned}$$

From the argument used in the theory of regenerative process, the pointwise yAO are seen to satisfy the following recursive relations:

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t) \alpha A_1(t) \\ A_1(t) &= M_1(t) + q_{1,3c}(t) \alpha A_{3c}(t) + \sum q_{1,4c}(t) \alpha A_{4c}(t) \\ A_{3c}(t) &= M_{3c}(t) + \sum q_{3c,6c}(t) \alpha A_{6c}(t) + \sum q_{3c,5c}(t) \alpha A_{5c}(t) \\ A_{4c}(t) &= \sum q_{4c,6c}(t) \alpha A_{6c}(t) \\ A_{5c}(t) &= M_{5c}(t) + q_{5c,0}(t) \alpha A_0(t) + q_{5c,1}(t) \alpha A_1(t) \\ &\quad + \sum q_{5c,7c}(t) \alpha A_{7c}(t) + \sum q_{5c,8c}(t) \alpha A_{8c}(t) \\ A_{6c}(t) &= q_{6c,1}(t) \alpha A_1(t) + \sum q_{6c,8c}(t) \alpha A_{8c}(t) \end{aligned}$$

$$\begin{aligned} A_{7c}(t) &= M_{7c}(t) + q_{7c,c}(t) \alpha A_0(t) + q_{7c,1}(t) \alpha A_1(t) \\ A_{8c}(t) &= q_{8c,1}(t) \alpha A_1(t) \end{aligned} \dots (43-50)$$

Taking Laplace-transform of the above equations and solving for $M_0(s)$ by omitting the argument "s" for brevity, therefore we (Jet the steady state availability when the system initially starts operation from S_0 is

$$A_0(\infty) = \lim_{s \rightarrow 0} s \cdot A^*_0(s) = N_2(O) / D'_2(O) = N_2/D_2 \dots (51)$$

where

$$\begin{aligned} N_2 &= \mu_0 (1 - \sum P_{1,3c} P_{3c,5c} (P_{5c,1} + P_{5c,8c} + P_{5c,7c} P_{7c,1}) \\ &\quad - \sum (P_{1,3c} P_{3c,6c} + P_{1,4c})) + \mu_1 + \sum P_{1,3c} \mu_{3c} \\ &\quad + \sum P_{1,3c} P_{3c,5c} (\mu_{5c} + P_{5c,7c} \mu_{7c}) \end{aligned} \dots (52)$$

and

$$\begin{aligned} D_2 &= m_1 + \sum P_{1,3c} (m_{2c} + P_{3c,5c} (P_{5c,0} + P_{5c,7c} P_{7c,0}) \mu_0 \\ &\quad - m_{3c} + P_{5c,7c} m_{4c}) + \sum P_{1,4c} \mu_{4c} + \sum (P_{1,3c} P_{3c,6c} \\ &\quad + P_{1,4c}) \mu_{6c} + P_{6c,8c} \sum (P_{1,3c} (P_{3c,5c} P_{5c,8c} + P_{3c,6c}) \\ &\quad + P_{1,4c}) \mu_{8c} \end{aligned} \dots (53)$$

VIII. BUSY PERIOD ANALYSIS

According to the definition of $W_i(t)$, we have

$$\begin{aligned} W_1 &= \bar{F}(t) & ; & \quad W_{3c}(t) = \bar{G}_c(t) = W_{4c}(t) \\ W_{5c} &= R_c(t) = W_{6c}(t) & ; & \quad W_{7c}(t) = A_c(t) = W_{8c}(t) \end{aligned} \dots (54-57)$$

Defining $B_i(t) = P[\text{system initial?} \text{ from regenerative state } S_1 \text{ is under repair at epoch } t]$

the following relations among $B_i(t)$ hold good

$$\begin{aligned} B_0(t) &= q_{01}(t) \alpha B_1(t) \\ B_1(t) &= W_1(t) + \sum q_{1,3c}(t) \alpha B_{3c}(t) + \sum q_{1,4c}(t) \alpha B_{4c}(t) \\ B_{3c}(t) &= W_{3c}(t) + \sum q_{3c,5c}(t) \alpha B_{5c} + \sum q_{3c,6c}(t) \alpha B_{6c}(t) \\ B_{4c}(t) &= W_{4c}(t) + \sum q_{4c,6c}(t) \alpha B_{6c}(t) \\ B_{5c}(t) &= W_{5c}(t) + q_{5c,0}(t) \alpha B_0(t) + q_{5c,1}(t) \alpha B_1(t) \\ &\quad + \sum q_{5c,7c}(t) \alpha B_{7c}(t) + \sum q_{5c,8c}(t) \alpha B_{8c}(t) \\ B_{6c}(t) &= W_{6c}(t) + q_{6c,1}(t) \alpha B_1(t) + \sum q_{6c,8c}(t) \alpha B_{8c}(t) \\ B_{7c}(t) &= W_{7c}(t) + q_{7c,0}(t) \alpha B_0(t) + q_{7c,1}(t) \alpha B_1(t) \\ B_{8c}(t) &= W_{8c}(t) + q_{8c,1}(t) \alpha B_1(t) \end{aligned} \dots (58-65)$$

Taking Laplace transform of the above equations and solving for $B^*_0(s)$ by omitting the argument "s" for brevity, one gets the fraction of time for which the system is under repair is given by

$$B_0 = \lim_{t \rightarrow \infty} B_0(t) = \lim_{s \rightarrow 0} s.B^*_0(s) = N_3 / D_2 \quad \dots (66)$$

where D_2 is same as in (53) and

$$N_3 = m_1 + \sum (m_{2c} + m_{3c}) + \sum m_{4c} (P_{1,3c} P_{3c,5c} (P_{5c,7c} + P_{5c,8c})) \quad (6c)$$

$$+ (P_{1,3c} P_{3c,6c} + P_{1,4c} P_{6c,8c}) \quad \dots (67)$$

IX. . EXPECTED NUMBER VISITS BY THE REPAIRMAN

Using the definition of $V_i(t)$ the following recursive relations among $V_i(t)$ can be obtained :

$$V_0(t) = Q_{01}(t) [1 + V_1(t)]$$

$$V_1(t) = \sum Q_{1,3c}(t) V_{3c}(t) + Q_{1,4c}(t) V_{4c}(t) \quad (2)$$

$$V_{3c}(t) = \sum Q_{3c,5c}(t) V_{5c}(t) + \sum Q_{3c,6c}(t) V_{6c}(t) \quad (4c)$$

$$V_{4c}(t) = \sum Q_{4c,6c}(t) V_{6c}(t)$$

$$V_{5c}(t) = Q_{5c,0}(t) V_0(t) + Q_{6c,1}(t) V_1(t)$$

$$+ \sum Q_{5c,7c}(t) V_{7c}(t) + \sum Q_{5c,8c}(t) V_{8c}(t) \quad (6c)$$

$$V_{7c}(t) = Q_{7c,0}(t) V_0(t) + Q_{7c,1}(t) V_1(t) \quad (8c)$$

$$V_{8c}(t) = Q_{8c,1}(t) V_1(t) \quad \dots (68)$$

Taking Laplace-Stieltjes transform of the above equations and solving for $V_0(s)$ by omitting the argument "s" for brevity, we get the expected number of visits per unit of time in steady state as-

$$V_0 = \lim_{t \rightarrow \infty} [V_0(t) / t] = \lim_{s \rightarrow 0} \tilde{V}_0(s) = N_4 / D_2$$

where D_2 is same as in (53) and

$$N_4 = \sum P_{1,3c} P_{3c,5c} (P_{5c,0} + P_{5c,7c} P_{7c,0})$$

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