

# INTUITIONISTIC FUZZY QUASI RGA-OPEN AND INTUITIONISTIC FUZZY QUASI RGA -CLOSED MAPPINGS

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**Abstract:** An intuitionistic fuzzy set  $A$  is said to be intuitionistic fuzzy rga-closed [21] in intuitionistic fuzzy topological spaces if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is intuitionistic fuzzy regular  $\alpha$ -open in  $X$ . In this paper we introduce quasi rga -open mapping from intuitionistic fuzzy topological space  $X$  to intuitionistic fuzzy topological  $Y$  as the image of every intuitionistic fuzzy rga -open set is intuitionistic fuzzy open. Also we obtain its Characterization and basic properties.

**Keywords:** Intuitionistic fuzzy rga -closed set, intuitionistic fuzzy rga -open sets, intuitionistic fuzzy rga -continuous mappings, intuitionistic fuzzy quasi rga -open mappings, intuitionistic fuzzy quasi rga -closed mappings.

## I. INTRODUCTION

Mapping plays an important role in study of modern mathematics, especially in Topology and Functional Analysis. Closed and open mapping are one such mapping which are studied for different type of closed sets by various mathematicians for the past many years. . The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. In 2008, Thakur and Chaturvedi [14] introduced the notion of intuitionistic fuzzy generalized closed set in intuitionistic fuzzy topological space. Since then different mathematicians worked and studied in different forms of intuitionistic fuzzy g-closed set and related topological properties. The authors of the paper introduced the concepts of intuitionistic fuzzy sg-closed sets [7] , intuitionistic fuzzy sg-continuous mappings[17], intuitionistic fuzzy sg-irresolute mappings[18], intuitionistic fuzzy rw-closed sets[20], intuitionistic fuzzy w-closed sets[19], intuitionistic fuzzy rga-closed sets[21], intuitionistic fuzzy rga continuity [8] , intuitionistic fuzzy gpr-closed sets[22], intuitionistic fuzzy gpr open and gpr-closed mappings [9] ,

intuitionistic fuzzy g-open and g-closed mappings [16] , intuitionistic fuzzy sg-open and sg-closed mappings [11] in intuitionistic fuzzy topology. In this paper we will continue the study of related concepts by involving intuitionistic fuzzy rga-open sets. We introduce and characterize the concepts of intuitionistic fuzzy quasi rga- -open and intuitionistic fuzzy quasi rga--closed mappings.

## II. PRELIMINARIES

**Definition 2.1:** An intuitionistic fuzzy set  $A$  of an intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is called:

- Intuitionistic fuzzy g-closed if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy open.[14]
- Intuitionistic fuzzy g-open if its complement  $A^c$  is intuitionistic fuzzy g-closed.[14]
- Intuitionistic fuzzy sg-closed if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy regular open.[7]
- Intuitionistic fuzzy sg-open if its complement  $A^c$  is intuitionistic fuzzy rg-closed.[7]
- Intuitionistic fuzzy w-closed if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy semi open.[19]
- Intuitionistic fuzzy w -open if its complement  $A^c$  is intuitionistic fuzzy w-closed.[19]
- Intuitionistic fuzzy rw-closed if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy regular semi open.[20]
- Intuitionistic fuzzy rw -open if its complement  $A^c$  is intuitionistic fuzzy rw-closed.[20]
- Intuitionistic fuzzy gpr-closed if  $pcl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy regular open.[22]
- Intuitionistic fuzzy gpr -open if its complement  $A^c$  is intuitionistic fuzzy gpr-closed.[22]
- Intuitionistic fuzzy rga -closed if  $\alpha cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy regular alpha open.[21]
- Intuitionistic fuzzy rga-open if its complement  $A^c$  is intuitionistic fuzzy rga-closed.[21]

**Definition 2.2 [21]** The  $rg\alpha$  - interior and  $rg\alpha$  - closure of an intuitionistic fuzzy set  $A$  of an intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  respectively denoted by  $rg\text{aint}(A)$  and  $rg\text{acl}(A)$  are defined as follows:

$rg\alpha \text{ int}(A) = \cup \{ V : V \subseteq A, V \text{ is intuitionistic fuzzy } rg\alpha \text{ - open} \}$

$rg\alpha \text{ cl}(A) = \cap \{ F : A \subseteq F, F \text{ is intuitionistic fuzzy } rg\alpha \text{ - closed} \}$

**Definition 2.3 [6]:** Let  $(X, \mathfrak{T})$  and  $(Y, \sigma)$  be two intuitionistic fuzzy topological spaces and let  $f: X \rightarrow Y$  be a function. Then  $f$  is said to be

- Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of  $Y$  is an intuitionistic fuzzy open set in  $X$ .
- Intuitionistic fuzzy closed if the image of each intuitionistic fuzzy closed set in  $X$  is an intuitionistic fuzzy closed set in  $Y$ .
- Intuitionistic fuzzy open if the image of each intuitionistic fuzzy open set in  $X$  is an intuitionistic fuzzy open set in  $Y$ .

**Definition 2.4:** Let  $(X, \mathfrak{T})$  and  $(Y, \sigma)$  be two intuitionistic fuzzy topological spaces and let  $f: X \rightarrow Y$  be a function. Then  $f$  is said to be

- Intuitionistic fuzzy  $g$ -continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy  $g$ -closed in  $X$ . [15]
- Intuitionistic fuzzy  $g$ -open if image of every open set of  $X$  is intuitionistic fuzzy  $g$ -open in  $Y$ . [16]
- Intuitionistic fuzzy  $g$ -closed if image of every closed set of  $X$  is intuitionistic fuzzy  $g$ -closed in  $Y$ . [16]
- Intuitionistic fuzzy  $sg$ -continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy  $sg$ -closed in  $X$ . [17]
- Intuitionistic fuzzy  $sg$ -open if image of every open set of  $X$  is intuitionistic fuzzy  $sg$ -open in  $Y$ . [10]
- Intuitionistic fuzzy  $sg$ -closed if image of every closed set of  $X$  is intuitionistic fuzzy  $sg$ -closed in  $Y$ . [10]
- Intuitionistic fuzzy  $sg$ -irresolute if the pre image of every intuitionistic fuzzy  $sg$ -closed set in  $Y$  is intuitionistic fuzzy  $sg$ -closed in  $X$ . [18]
- Intuitionistic fuzzy  $w$ -continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy  $w$ -closed in  $X$ . [19]
- Intuitionistic fuzzy  $w$ -open if image of every open set of  $X$  is intuitionistic fuzzy  $w$ -open in  $Y$ . [19]
- Intuitionistic fuzzy  $w$ -closed if image of every closed set of  $X$  is intuitionistic fuzzy  $w$ -closed in  $Y$ . [19]
- Intuitionistic fuzzy  $gpr$ -continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy  $gpr$ -closed in  $X$ . [22]
- Intuitionistic fuzzy  $gpr$ -open if image of every open set of  $X$  is intuitionistic fuzzy  $gpr$ -open in  $Y$ . [9]
- Intuitionistic fuzzy  $gpr$ -closed if image of every closed set of  $X$  is intuitionistic fuzzy  $gpr$ -closed in  $Y$ . [9]

- Intuitionistic fuzzy  $rg\alpha$ -continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy  $rg\alpha$ -closed in  $X$ . [8]
- Intuitionistic fuzzy  $rg\alpha$ -open if image of every open set of  $X$  is intuitionistic fuzzy  $rg\alpha$ -open in  $Y$ . [8]
- Intuitionistic fuzzy  $rg\alpha$ -closed if image of every closed set of  $X$  is intuitionistic fuzzy  $rg\alpha$ -closed in  $Y$ . [8]

**Definition 2.5:** [10] A mapping  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy quasi  $rg\alpha$ -open if the image of every intuitionistic fuzzy  $rg\alpha$ -open set of  $X$  is intuitionistic fuzzy open set in  $Y$ .

**Definition 2.6:** [10] A mapping  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy quasi  $rg\alpha$ -closed if image of every intuitionistic fuzzy  $rg\alpha$ -closed set of  $X$  is intuitionistic fuzzy closed set in  $Y$ .

### III. INTUITIONISTIC FUZZY QUASI $rg\alpha$ -OPEN MAPPINGS.

**Definition 3.1:** A mapping  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy quasi  $rg\alpha$ -open if the image of every intuitionistic fuzzy  $rg\alpha$ -open set of  $X$  is intuitionistic fuzzy open set in  $Y$ .

**Theorem 3.1:** A mapping  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy quasi  $rg\alpha$ -open if and only if for every intuitionistic fuzzy set  $U$  of  $X$   $f(rg\alpha \text{ int}(U)) \subseteq \text{int}(f(U))$ .

**Proof: Necessity** Let  $f$  be an intuitionistic fuzzy quasi  $rg\alpha$ -open mapping and  $U$  is an intuitionistic fuzzy open set in  $X$ . Now  $\text{int}(U) \subseteq U$  which implies that  $f(\text{int}(U)) \subseteq f(U)$ . Since  $f$  is an intuitionistic fuzzy quasi  $rg\alpha$ -open mapping,  $f(rg\alpha \text{ int}(U))$  is intuitionistic fuzzy open set in  $Y$  such that  $f(rg\alpha \text{ int}(U)) \subseteq f(U)$  therefore  $f(rg\alpha \text{ int}(U)) \subseteq \text{int}(f(U))$ .

**Sufficiency:** For the converse suppose that  $U$  is an intuitionistic fuzzy  $rg\alpha$ -open set of  $X$ . Then  $f(U) = f(rg\alpha \text{ int}(U)) \subseteq \text{int}(f(U))$ . But  $\text{int}(f(U)) \subseteq f(U)$ . Consequently  $f(U) = \text{int}(f(U))$  which implies that  $f(U)$  is an intuitionistic fuzzy open set of  $Y$  and hence  $f$  is an intuitionistic fuzzy quasi  $rg\alpha$ -open mapping.

**Theorem 3.2:** If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is an intuitionistic fuzzy quasi  $rg\alpha$ -open mapping then  $rg\alpha \text{ int}(f^{-1}(G)) \subseteq f^{-1}(\text{int}(G))$  for every intuitionistic fuzzy set  $G$  of  $Y$ .

**Proof:** Let  $G$  is an intuitionistic fuzzy set of  $Y$ . Then  $rg\alpha \text{ int}(f^{-1}(G))$  is an intuitionistic fuzzy  $rg\alpha$ -open set in  $X$ . Since  $f$  is intuitionistic fuzzy quasi  $rg\alpha$ -open  $f(rg\alpha \text{ int}(f^{-1}(G)))$  is intuitionistic fuzzy open in  $Y$  and hence by theorem 3.1  $f(rg\alpha \text{ int}(f^{-1}(G))) \subseteq \text{int}(f(f^{-1}(G))) \subseteq \text{int}(G)$  Thus  $rg\alpha \text{ int}(f^{-1}(G)) \subseteq f^{-1}(\text{int}(G))$ .

**Theorem 3.3:** A mapping  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy quasi  $rg\alpha$ -open if and only if for each intuitionistic fuzzy set  $S$  of  $Y$  and for each intuitionistic fuzzy  $rg\alpha$ -closed set  $U$  of  $X$  containing  $f^{-1}(S)$  there is an intuitionistic fuzzy closed  $V$  of  $Y$  such that  $S \subseteq V$  and  $f^{-1}(V) \subseteq U$ .

**Proof: Necessity:** Suppose that  $f$  is an intuitionistic fuzzy quasi  $rg\alpha$ -open mapping. Let  $S$  be the intuitionistic fuzzy set

of  $Y$  and  $U$  is an intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of  $X$  such that  $f^{-1}(S) \subseteq U$ . Then  $V = (f^{-1}(U^c))^c$  is intuitionistic fuzzy closed set of  $Y$  such that  $f^{-1}(V) \subseteq U$ .

**Sufficiency:** Suppose that  $F$  is an intuitionistic fuzzy  $\text{rg}\alpha$ -open set of  $X$ . Then  $f^{-1}((f(F))^c) \subseteq F^c$  and  $F^c$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed set in  $X$ . By hypothesis there is an intuitionistic fuzzy closed set  $V$  of  $Y$  such that  $(f(F))^c \subseteq V$  and  $f^{-1}(V) \subseteq F^c$ . Therefore  $F \subseteq (f^{-1}(V))^c$ . Hence  $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$  which implies  $f(F) = V^c$ . Since  $V^c$  is intuitionistic fuzzy open set of  $Y$ . Hence  $f(F)$  is intuitionistic fuzzy open in  $Y$  and thus  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -open mapping.

**Theorem 3.4:** An intuitionistic fuzzy mapping  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is an intuitionistic fuzzy quasi  $\text{rg}\alpha$ -open mapping if and only if  $f^{-1}(\text{cl}(B)) \subseteq \text{rg}\alpha \text{cl}(f^{-1}(B))$  for every intuitionistic fuzzy set  $B$  of intuitionistic fuzzy topological space  $Y$ .

**Proof: Necessity:** Suppose that  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -open for any intuitionistic fuzzy set  $B$  of intuitionistic fuzzy topological space  $Y$ . Now  $\text{rg}\alpha \text{cl}(f^{-1}(B))$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed set in  $X$  such that  $f^{-1}(\text{cl}(B)) \subseteq \text{rg}\alpha \text{cl}(f^{-1}(B))$ . Therefore by theorem 3.3 there exists an intuitionistic fuzzy closed set  $F$  in  $Y$  such that  $B \subseteq F$  and  $f^{-1}(F) \subseteq \text{rg}\alpha \text{cl}(f^{-1}(B))$ . Therefore we obtain  $f^{-1}(\text{cl}(B)) \subseteq f^{-1}(F) \subseteq \text{rg}\alpha \text{cl}(f^{-1}(B))$ .

**Sufficiency:** Let  $B \subseteq Y$  and  $F$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of  $X$  containing  $f^{-1}(B)$ . Put  $W = \text{cl}(B)$  then we have  $B \subseteq W$  and  $W$  is intuitionistic fuzzy closed set in  $Y$  such that  $f^{-1}(W) \subseteq \text{rg}\alpha \text{cl}(f^{-1}(B)) \subseteq F$ . Then by theorem 3.3  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -open.

**Theorem 3.5:** If  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \mu)$  be two intuitionistic fuzzy mappings and  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -open. If  $g$  is intuitionistic fuzzy continuous then  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -open.

**Proof:** Let  $U$  be an intuitionistic fuzzy  $\text{rg}\alpha$ -open set in  $X$  then  $g \circ f(U)$  is open in  $Z$ . Since  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -open mapping. Again  $g : (Y, \sigma) \rightarrow (Z, \mu)$  is intuitionistic fuzzy continuous and  $g \circ f(U)$  is intuitionistic fuzzy open in  $Z$ . Therefore  $g^{-1}(g \circ f(U)) = f(U)$  is intuitionistic fuzzy open in  $X$ . This shows that  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -open mapping.

#### IV. INTUITIONISTIC FUZZY QUASI $\text{rg}\alpha$ -CLOSED MAPPINGS.

**Definition 4.1:** A mapping  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed if image of every intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of  $X$  is intuitionistic fuzzy closed set in  $Y$ .

**Theorem 4.1:** A mapping  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed if and only if for each intuitionistic fuzzy set  $S$  of  $Y$  and for each intuitionistic fuzzy  $\text{rg}\alpha$ -open set  $U$  of  $X$  containing  $f^{-1}(S)$  there is an intuitionistic fuzzy open set  $V$  of  $Y$  such that  $S \subseteq V$  and  $f^{-1}(V) \subseteq U$ .

**Proof Necessity:** Suppose that  $f$  is an intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed mapping. Let  $S$  be the intuitionistic fuzzy closed set of  $Y$  and  $U$  is an intuitionistic fuzzy  $\text{rg}\alpha$ -open set of  $X$  such that  $f^{-1}(S) \subseteq U$ . Then  $V = Y - f^{-1}(U^c)$  is intuitionistic fuzzy open set of  $Y$  such that  $f^{-1}(V) \subseteq U$ .

**Sufficiency:** For the converse suppose that  $F$  is an intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of  $X$ . Then  $(f(F))^c$  is an intuitionistic fuzzy set of  $Y$  and  $F^c$  is intuitionistic fuzzy  $\text{rg}\alpha$ -open set in  $X$  such that  $f^{-1}((f(F))^c) \subseteq F^c$ . By hypothesis there is an intuitionistic fuzzy open set  $V$  of  $Y$  such that  $(f(F))^c \subseteq V$  and  $f^{-1}(V) \subseteq F^c$ . Therefore  $F \subseteq (f^{-1}(V))^c$ . Hence  $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$  which implies  $f(F) = V^c$ . Since  $V^c$  is intuitionistic fuzzy closed set of  $Y$ . Hence  $f(F)$  is intuitionistic fuzzy closed in  $Y$  and thus  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed mapping.

**Theorem 4.2:** If  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed mapping and  $g : (Y, \sigma) \rightarrow (Z, \mu)$  are intuitionistic fuzzy closed map. Then  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed mapping.

**Proof:** Let  $H$  be an intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of intuitionistic fuzzy topological space  $(X, \mathfrak{I})$ . Then  $f(H)$  is intuitionistic fuzzy closed set of  $(Y, \sigma)$  because  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed mapping. Now  $(g \circ f)(H) = g(f(H))$  is intuitionistic fuzzy closed set in intuitionistic fuzzy topological space  $Z$  because  $g$  is intuitionistic fuzzy closed map. Thus  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed mapping.

**Theorem 4.3:** If  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed mapping and  $g : (Y, \sigma) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed map. Then  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed.

**Proof:** Let  $H$  be an intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of intuitionistic fuzzy topological space  $(X, \mathfrak{I})$ . Then  $f(H)$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of  $(Y, \sigma)$  because  $f$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed mapping. Now  $(g \circ f)(H) = g(f(H))$  is intuitionistic fuzzy closed set in intuitionistic fuzzy topological space  $Z$  because  $g$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed mapping. Thus  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed.

**Definition 4.2:** A mapping  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy  $\text{rg}\alpha^*$ -closed if image of every intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of  $X$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed set in  $Y$ .

**Theorem 4.4:** Let  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \mu)$  are intuitionistic fuzzy mapping. Then

- If  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed and  $g$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed, then  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy  $\text{rg}\alpha^*$ -closed.
- If  $f$  is intuitionistic fuzzy  $\text{rg}\alpha^*$ -closed and  $g$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed, then  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed.

**Proof: (a)** Let  $H$  be an intuitionistic fuzzy  $\text{rg}\alpha$ -closed set of intuitionistic fuzzy topological space  $(X, \mathfrak{I})$ . Then  $f(H)$  is intuitionistic fuzzy closed set of  $(Y, \sigma)$  because  $f$  is intuitionistic fuzzy quasi  $\text{rg}\alpha$ -closed mapping. Now  $(g \circ f)(H) = g(f(H))$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed set in intuitionistic fuzzy topological space  $Z$  because  $g$  is intuitionistic fuzzy  $\text{rg}\alpha$ -closed mapping. Thus  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy  $\text{rg}\alpha^*$ -closed.

(b) Let  $H$  be an intuitionistic fuzzy  $rg\alpha$ -closed set of intuitionistic fuzzy topological space  $(X, \mathfrak{I})$ . Then  $f(H)$  is intuitionistic fuzzy  $rg\alpha$ -closed set of  $(Y, \sigma)$  because  $f$  is intuitionistic fuzzy  $rg\alpha^*$ -closed map. Now  $(g \circ f)(H) = g(f(H))$  is intuitionistic fuzzy closed set in intuitionistic fuzzy topological space  $Z$  because  $g$  is intuitionistic fuzzy quasi  $rg\alpha$ -closed mapping. Thus  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $rg\alpha$ -closed.

**Theorem 4.5 :** Let  $f : (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \mu)$  are intuitionistic fuzzy mapping such that  $g \circ f : (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy quasi  $rg\alpha$ -closed. If  $g$  is intuitionistic fuzzy  $rg\alpha$ -continuous then  $f$  is intuitionistic fuzzy  $rg\alpha^*$ -closed.

**Proof :** Suppose  $H$  is intuitionistic fuzzy  $rg\alpha$ -closed set in  $X$ . Since  $g \circ f$  is intuitionistic fuzzy quasi  $rg\alpha$ -closed  $(g \circ f)(H)$  is intuitionistic fuzzy closed in  $Z$ . Since  $g$  is intuitionistic fuzzy  $rg\alpha$ -continuous  $g^{-1}((g \circ f)(H)) = f(H)$  is intuitionistic fuzzy  $rg\alpha$ -closed in  $Y$ . Hence  $f$  is intuitionistic fuzzy  $rg\alpha^*$ -closed.

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