International Journal of Technical Research and Applications e-ISSN: 2320-8163,
www.ijtra.com Volume 4, Issue 3 (May-June, 2016), PP. 356-360

CONNECTIVITY IN UNCERTAIN GRAPHS (CUG) – EXAMINING USING DISTRIBUTION FUNCTION

P.Kamal Devi[1], G.Eswara Prasad[2], V.S.Mathu Suresh[3]
[1]Assistant Professor, [2]Head of the Department, [3]Associate professor
Department of Mathematics
[1]Pioneer Kumaraswamy College, Nagercoil
[2]S.T. Hindu College, Nagercoil, Kanyakumari District-Tamil Nadu, India
[3]ROHINI College of Engineering and Technology, Near Anjugramam Junction, Palkulam, Variyoor Post, Kanyakumari District.
kdeviprasad2011@gmail.com
geprasad2002@yahoo.com

Abstract— **The emerging network applications, querying and mining from the uncertain graphs has become increasingly important. There is a growing need for methods that can represent and query about the uncertain graphs. These uncertain graphs are often the result of an information extraction and integration system that attempts to extract an entity graph or a knowledge graph from multiple unstructured sources. Such integration typically leads to identity uncertainty, as different data sources may use different references to the same underlying real-world entities. In uncertain graphs, the existence of some edges is not predetermined. The connectivity of an uncertain graph is essentially an uncertain variable, which indicates the suitability for investigation of its distribution function. The main focus of this paper is to propose a framework to determine the distribution function of the connectivity of an uncertain graph. Initially, it focus on the discussion of the characteristics of the uncertain connectivity and the distribution function is derived. An efficient algorithm is designed based on Floyd's algorithm that depicts the connectivity parameters can also be focused to improve the network performance. .**

Index terms- **Uncertainty, Uncertain graphs, Connectivity, Distribution function and information extraction.**

I. INTRODUCTION

To the best of our insight, in the traditional diagram hypothesis, the edges and vertices are predestined [1,2]. Hypothetical issues on chart hypothesis are concerned with integration, nature of the diagram and determination of width. To take care of these issues, an assortment of proficient calculations have been proposed in the course of the most recent decades and effectively connected to a lot of people certifiable issues, for example, transportation, interchanges, and store network administration. In practice, indeterminacy is unavoidable because of the non-existence of data. The traditional calculations are given off an impression of being exceptionally hard to apply straightforwardly the indeterminacy in appreciation of vertices and edges. In this paper, we have considered the presence of some non-deterministic edges. The presence of such non-deterministic charts is utilized to depict the assembly of a system [3,4].

To manage non-deterministic diagrams, a few analysts presented likelihood hypothesis and created arbitrary charts. Normally, an E-R arbitrary chart is acquired by beginning with a set of n secluded vertices and including progressive edges between them with likelihood $0 < p < 1$. In this paper, we first study the attributes of width in an indeterminate chart, and after that get the comparing appropriation capacity. Besides, we expressly plan a calculation got from the Floyd's calculation to figure the circulation capacity [5,6]. The productivity of the calculation is at long last demonstrated hypothetically and tentative. The uncertain theory was designed in 2007 and re-designed by Liu [21,22]. "Uncertainty Theory", second ed., Springer-Verlag, Berlin.] is a great tool to deal with the non-deterministic resources with the advent of professional data. Nowadays, the concept of uncertain theory has been deployed in the network optimization, inventory problem transportation problem. In 2013, [13] introduced the concepts of uncertain theory into the graph theory. In their research, they introduced the connected index for the uncertain graph. The connectedness indexes were formulated on the use of Kruskal's [15] and Prim's [16] algorithm. To the best of our knowledge, there is no related study on the connectivity of the uncertain graphs [8,9]. This paper focuses on the connectivity of the uncertain graphs which is a tentative variable due to the presence of the uncertain edges. In this paper, we first study the characteristics of connectivity in an uncertain graph, and then obtain the corresponding distribution function.

The remainder of this paper is organized as follows. In Section 1, an introduction about the graph theory. In Section 2, uncertainty theory is introduced briefly for the completeness of this research. In Section 3, the distribution function of the connectivity of uncertain graph is obtained. In Section 4, an efficient framework for calculating the distribution function is proposed and illustrated with some numerical examples. In section 5, the conclusion of the research.

II. UNCERTAINTY THEORY

Let T be a non empty set and μ be algebra over T. Each element $\Lambda \in \mu$ is assigned a number $M\{\Lambda\}$. In order to ensure the mathematical properties [15,12,17], Liu presented the following three axioms:

Axiom 1: The normality of a graph is predicted as $M\{T\} = 1$

Axiom 2: The duality of a graph is predicted as $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for all the event Λ.

Axiom 3: The sub additivity for every event $M\{\Lambda_i\}$ stated as: $M\{\Lambda_i = 1\} \le \sum_{i=1} M\{\Lambda_i\}$

Definition 1: [Liu] The set function *M* is called an uncertain measure, if it meets the condition of normality, self-duality and sub additivity axioms. The triplet function *(T, μ, M)* is called the uncertain space. The uncertain measure *(T, μ, M)* is the chance that every event occurs. The product uncertain measure was studied by [22] which takes to the next step calls 'product measure axiom'.

Axiom 4: Let (T_i, μ_i, M_i) be the uncertainty spaces for every *i= 1, 2, 3…….n*. The product uncertain measure, M is an uncertain measure satisfying

$$M\left\{\prod_{i=1}^{\infty} \Lambda_i\right\} = \bigcap_{i=1}^{\infty} M\{\Lambda_i\}$$

Definition 2: The tentative variable is a measurable function n from an uncertain space *(T, μ, M)* to be the set of real numbers.

$$\{\xi_i \in B\} = \{\gamma \in T \mid \xi(\gamma) \in B\}$$

Definition 3: The uncertain variables is said to be independent if it follows

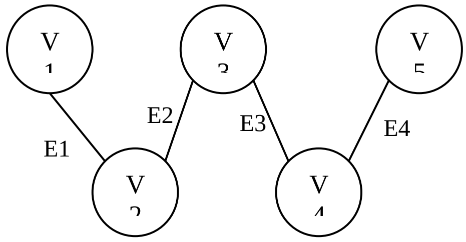
$$M\left\{\bigcap_{i=1}^{\infty}\{\xi_i \in B_i\}\right\} = \min_{1\le i\le n} M\{\xi_i \in B_i\}$$

III. UNCERTAIN GRAPH AND ITS CONNECTIVITY

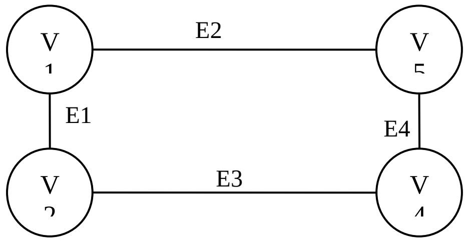
A. Uncertain graph concepts:

In this paper, the terminologies associated with this related to the illustration from [25].

Definition 1: A graph G is a triple consisting of a vertex set *V(G)* and edge set *E(G)* (fig. 1) and the relation between the two edges is called endpoints [20]. The endpoints are equal when the edge is a loop. The endpoints are similar in the nature when they are having multiple edges. The number of the vertices in the *G* is called as the order of *G*.



$G_1 = (V_1, E_1)$



$G_1 = (V_1, E_1)$

Fig. 1 Grpah with Edge Set and Vector Set

Consider a graph of order 4, and then the adjacency matrix is known as

$$A = \begin{Bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{Bmatrix}$$

Where,

$$a_{ij} = \begin{Bmatrix} 1 & if\ i\ and\ j\ are\ the\ endpoints\ of\ the\ edge \\ 0 & otherwise \end{Bmatrix}$$

Definition 2: In a graph *G*, a walk is the list of V_0, $E_1, \dots E_k$, V_k of the vertices and edges such that for $1< i<k$. The walk from the V_1 to V_n possesses no repeated vertex.

Definition 3: A graph *G* is connected if there is a *u–v* path whenever $u, v \in V(G)$

3.2 Connectivity in an uncertain graph

The connectivity is a basic concept in graph theory, which measures the connection between the vertices. The connectivity is given as:

$$G(V,E) = conG(\min_{v_i, v_j} \in Vd(v_i, v_j))$$

The distribution function of connectivity is given as:

$$M\{con(\varphi_1 \dots \varphi_m) \le K\} = \begin{cases} \sup\limits_{(B_1,\dots B_m)\in(-\infty,k]} \min\limits_{1\le i\le m} M\{\xi_i \in B_i\} & if \sup\limits_{(B_1,\dots B_m)\in(-\infty,k]} \min\limits_{1\le i\le m} M\{\xi_i \in B_i\} > 0.5 \\ 1 - \sup\limits_{(B_1,\dots B_m)\in[k,+\infty)} \min\limits_{1\le i\le m} M\{\xi_i \in B_i\} & if \sup\limits_{(B_1,\dots B_m)\in[k,+\infty)} \min\limits_{1\le i\le m} M\{\xi_i \in B_i\} > 0.5 \\ 0.5 & otherwise \end{cases}$$

IV. CONNECTIVITY IN AN UNCERTAIN GRAPH WITH AN EXAMPLE

According to the section 3.2 distribution function of the connectivity is given as $M\{conG \le k\}$ in an uncertain graph has to traverse the set. The algorithm for finding the connectivity of an uncertain graph using Floyd's algorithm is depicted as follows:

1. Sort the set $(x_1,\ x_2, \ldots x_m)$ in descending order. Without the loss of normality, it is assumed that $1 = x_0 \ge x_1 > x_2 > x_{n+1} = 0$. Set $j = 1$
2. In uncertain graph *G*, remove the pair $G(Vi, Ei)$ that satisfies $x_i < x_j$ where $i = 1,\ 2..\ m$. The graph (fig. 2) is modified according to the condition and new graph is calculated in G_j.
3. Denote the adjacent graph by G_j as G_j^*
4. Calculate $conG_j^* : setj = j+1$ and again repeat the process 2 still the shortest path is found.

The working of algorithm is as follows:

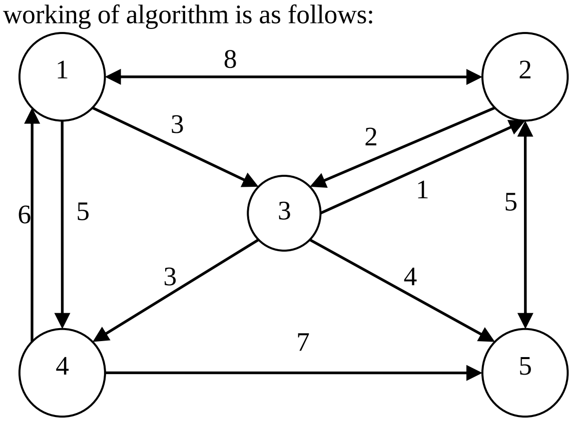


Fig. 2 Connectivity in an Uncertain Graph

The matrix is given as:

$$D_o = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 8 & 3 & 5 & \infty \\ 2 & 8 & 0 & 2 & \infty & 5 \\ 3 & 0 & 1 & 0 & 3 & 4 \\ 4 & 6 & \infty & \infty & 0 & 7 \\ 5 & \infty & 5 & \infty & \infty & 0 \end{array}$$

The starting matrix is given as:

$$Q_o = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 1 & 1 & 1 & 1 \\ 2 & 2 & - & 2 & 2 & 2 \\ 3 & 3 & 3 & - & 3 & 3 \\ 4 & 4 & 4 & 4 & - & 4 \\ 5 & 5 & 5 & 5 & 5 & - \end{array}$$

The first step is as follows let *i*=1, the calculation is as follows:

$$C_{1,2}{}^1 = min\left(c_{1,2}{}^0;\ d_{1,1}{}^0 + c_{1,2}{}^0\right) = min(8; 0+8) = 8$$

$$C_{1,3}{}^1 = min\left(c_{1,3}{}^0;\ d_{1,1}{}^0 + c_{1,3}{}^0\right) = min(3; 0+3) = 3$$

$$C_{1,4}{}^1 = min\left(c_{1,4}{}^0;\ d_{1,1}{}^0 + c_{1,4}{}^0\right) = min(5; 0+5) = 5$$

$$C_{1,5}{}^1 = min\left(c_{1,5}{}^0;\ d_{1,1}{}^0 + c_{1,5}{}^0\right) = min(\infty; 0+\infty) = \infty$$

$$C_{2,1}{}^1 = min\left(c_{2,1}{}^0;\ d_{2,1}{}^0 + c_{1,1}{}^0\right) = min(8; 8+0) = 8$$

$$C_{2,3}{}^1 = min\left(c_{2,3}{}^0;\ d_{2,1}{}^0 + c_{1,3}{}^0\right) = min(2; 8+3) = 2$$

Similarly, the same step is followed until the shortest path is found. The adjacency matrix is calculated as:

$$D_o = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 8 & 3 & 5 & \infty \\ 2 & 8 & 0 & 2 & \infty & 5 \\ 3 & 0 & 1 & 0 & 3 & 4 \\ 4 & 6 & \infty & \infty & 0 & 7 \\ 5 & \infty & 5 & \infty & \infty & 0 \end{array}$$

$$Q_o = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 1 & 1 & 1 & 1 \\ 2 & 2 & - & 2 & 2 & 2 \\ 3 & 3 & 3 & - & 3 & 3 \\ 4 & 4 & 4 & 4 & - & 4 \\ 5 & 5 & 5 & 5 & 5 & - \end{array}$$

$$D_1 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} 0 & 8 & 3 & 5 & \infty \\ 8 & 0 & 2 & 13 & 5 \\ 0 & 1 & 0 & 3 & 4 \\ 6 & 14 & 9 & 0 & 7 \\ \infty & 5 & \infty & \infty & 0 \end{bmatrix}}$$

$$D_3 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} 0 & 4 & 3 & 5 & 7 \\ 2 & 0 & 2 & 5 & 5 \\ 0 & 1 & 0 & 3 & 4 \\ 6 & 10 & 9 & 0 & 7 \\ 7 & 5 & 7 & 10 & 0 \end{bmatrix}}$$

$$Q_1 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} - & 1 & 1 & 1 & 1 \\ 2 & - & 2 & 2 & 2 \\ 3 & 3 & - & 3 & 3 \\ 4 & 1 & 1 & - & 4 \\ 5 & 5 & 5 & 5 & - \end{bmatrix}}$$

$$Q_3 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} - & 3 & 1 & 1 & 3 \\ 3 & - & 2 & 3 & 2 \\ 2 & 3 & - & 3 & 3 \\ 4 & 3 & 1 & - & 4 \\ 3 & 5 & 2 & 3 & - \end{bmatrix}}$$

$$D_2 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} 0 & 8 & 3 & 5 & 13 \\ 8 & 0 & 2 & 13 & 5 \\ 0 & 1 & 0 & 3 & 4 \\ 6 & 14 & 9 & 0 & 7 \\ 13 & 5 & 7 & 18 & 0 \end{bmatrix}}$$

$$D_4 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} 0 & 4 & 3 & 5 & 7 \\ 2 & 0 & 2 & 5 & 5 \\ 0 & 1 & 0 & 3 & 4 \\ 6 & 10 & 9 & 0 & 7 \\ 7 & 5 & 7 & 10 & 0 \end{bmatrix}}$$

$$Q_2 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} - & 1 & 1 & 1 & 2 \\ 2 & - & 2 & 2 & 2 \\ 3 & 3 & - & 3 & 3 \\ 4 & 1 & 1 & - & 4 \\ 2 & 5 & 2 & 2 & - \end{bmatrix}}$$

$$Q_4 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \overset{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}}{\begin{bmatrix} - & 3 & 1 & 1 & 3 \\ 3 & - & 2 & 3 & 2 \\ 2 & 3 & - & 3 & 3 \\ 4 & 3 & 1 & - & 4 \\ 3 & 5 & 2 & 3 & - \end{bmatrix}}$$

$$D_5 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 3 & 5 & 7 \\ 2 & 2 & 0 & 2 & 5 & 5 \\ 3 & 0 & 1 & 0 & 3 & 4 \\ 4 & 6 & 10 & 9 & 0 & 7 \\ 5 & 7 & 5 & 7 & 10 & 0 \end{array}$$

$$Q_5 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 3 & 1 & 1 & 3 \\ 2 & 3 & - & 2 & 3 & 2 \\ 3 & 2 & 3 & - & 3 & 3 \\ 4 & 4 & 3 & 1 & - & 4 \\ 5 & 3 & 5 & 2 & 3 & - \end{array}$$

The last matrix *D5* and *Q5* shows us the connectivity of the shortest paths. Hence from above graph the shortest path is found as 5 to 2 to 3 to 4 is the shortest path.

V. CONCLUSION

In this paper, we focused on the distribution function of the connectivity of the uncertain graphs. The algorithm is used to calculate the shortest path of a given uncertain graph. It has found that it follows the polynomial time complexity of the $O(mn^3)$, where m is the count of edges and n is the count of vertices. Moreover, in this paper the vertices are predetermined. It focus on the simulation of finding the shortest path by taking the parameter connectivity. In future, the research may investigate with the flow between the edges in an uncertain graph.

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