

RELAY SELECTION FOR AF BASED COOPERATIVE NETWORKS

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Abstract— In this paper, a performance analysis of amplify-and-forward (AF) relaying cooperative diversity system with relay selection is given when all terminals are moving. We have derived analytical expressions for probability density function (PDF), cumulative distribution function (CDF) and moment generating function (MGF) of end-to-end signal-to-noise ratio (SNR) for the considered system. Then, the symbol error rate performance (SER) of the system is calculated for MPSK modulation assuming the all links have double Rayleigh distribution.

Index Terms— Cooperative communication, Amplify-and-forward, Relay selection, Double Rayleigh fading

I. INTRODUCTION

In wireless communication channels, multipath fading is a major issue. Antenna diversity is an effective technique for mitigating channel fading. However, in most wireless applications, multiple antennas may not be useful due to the size, complexity and power limitations. Cooperative diversity is a technique which is proposed to combat the multipath fading [1]-[4]. The main idea behind cooperative diversity systems is that the source and the destination terminals communicate with each other with the help of intermediate nodes called relay terminals. There are two main cooperative relaying protocols: Amplify and Forward (AF) and Decode and Forward (DF). In AF technique, the relay terminal, retransmit the scaled version of the received signal without decoding the message. In DF protocol, relay decodes the received signal then re-encode and re-transmit it to the destination [5]-[7]. The fundamental destructive effect encountered in AF based wireless networks is the re-transmission of the amplified version of the noise terms while the most important problem in DF based cooperative systems is the error propagation due to the decoding errors at relay terminals which cause reduction in the effective signal-to-noise ratio (SNR) at the destination.

It has been shown that relay selection improves the performance of wireless relay networks [8]-[10]. In most of the relay selection-based cooperative network structures in the literature, the data transmission is realized with single relay selection. Zhao et al. proposed relay selection methods to improve the performance of AF based cooperative systems [8]. In [11], the authors propose a relay selection criterion in which the relay with the maximum SNR in relay-to-destination link cooperates. A performance analysis of cooperative diversity systems with best relay selection over Rayleigh fading channels is presented in [10]. In the literature, performance evaluation of AF based systems has been studied comprehensively. However, most of the literature are limited to Rayleigh fading channels, which are commonly used to characterize the cellular radio systems. Experimental results and theoretical analyzes have reported that Rayleigh fading channel model and the related

second-order channel statistics have been proposed for a base station-to-mobile link and these channels are insufficient for modeling dynamic mobile-to-mobile links [12]-[15].

Cascaded Rayleigh fading channel model has been presented in [16]-[20]. This model provides a realistic description of mobile-to-mobile channels where two or more independent Rayleigh fading processes are assumed to be generated by independent groups of scatterers around the two mobile terminals. *Cascaded* Rayleigh fading channel obtained by the product of N-independent Rayleigh distributed random variables. For N = 2, it reduces to *double* Rayleigh fading distribution [21]-[23]. In [24], theoretical analyzes have been presented for the cooperative relaying system which has the *double* Rayleigh distribution between source and destination terminals.

In this paper, we consider a mobile-to-mobile scenario and present the performance analyzes for AF based cooperative communication systems with relay selection. All channels are modeled as *double* Rayleigh fading. We derive exact symbol error rate (SER) expression based on moment generating function (MGF) for MPSK modulation. To do this, we obtain cumulative distribution function (CDF) and probability density function (PDF).

The reminder of this paper is organized as follows: In Section II system model is presented. We focus on the derivation of the PDFs and CDFs for the end-to-end SNR of considered system in Section III. Then, we investigate the SER of the system by using MGF expression. In Section IV, the numerical results are presented. Finally, some conclusions are given in Section V.

II. SYSTEM MODEL

We consider a multiple-relay cooperative communication system with a source (S), a destination (D), and N relay (R) nodes as shown in Figure 1. Every terminal operates in half-duplex mode and is equipped with a single transmit and receive antennas.

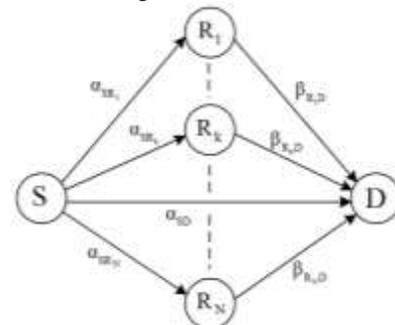


Fig. 1. System model.

The complex fading coefficients of source-to-kth relay (S→R_k), kth relay-to-destination (R_k→D) and source-to-

destination (S→D) links are denoted by α_{SR_k} , β_{R_kD} and α_{SD} , respectively. All of the links are modeled as the product of two independent complex Gaussian random variables [13] i.e., $\alpha_{SR_k} = \alpha_{SR_k}^1 \alpha_{SR_k}^2$, $\beta_{R_kD} = \beta_{R_kD}^1 \beta_{R_kD}^2$, $\alpha_{SD} = \alpha_{SD}^1 \alpha_{SD}^2$ where all variables have zero mean and variance of 0.5 per dimension. Therefore, the magnitudes of links $|\alpha_{SR_k}|$, $|\beta_{R_kD}|$, $|\alpha_{SD}|$ follow a *double* Rayleigh distribution [17], [18]

$$f(|\zeta|) = 4|\zeta|K_0(2|\zeta|) \quad (1)$$

where $K_0(\cdot)$ is the modified Bessel function of second kind of zero order [25].

In this paper, destination node is assumed to have full channel state information of the system via pilot signaling, determines the relay with best channel condition and declares the index of the selected relay to all relays through the feedback link. The whole transmission is accomplished in two phases. In the first phase, the source transmits MPSK (M-ary phase shift keying) symbol block x with unit power, while the relays and destination listen. The received signals by the relays and destination can be given as:

$$y_{SR_k} = \sqrt{E_s} x \alpha_{SR_k} + n_{SR_k}, \quad (2)$$

$$y_{SD} = \sqrt{E_s} x \alpha_{SD} + n_{SD}. \quad (3)$$

Where n_{SR_k} and n_{SD} are complex additive white Gaussian noises at the k^{th} relay and at the destination, respectively. E_s is the average energy per symbol.

In the second phase, the selected relay terminal retransmits a scaled version of received signal without decoding the message while the destination listens and the source keeps silent. The received signal at the destination is,

$$y_{R_kD} = y_{SR_k} \beta_{R_kD} G + n_{R_kD} \quad (4)$$

where n_{R_kD} is the complex additive white Gaussian noise

and $G = \left(\frac{\sqrt{E_s}}{\sqrt{E_s |\alpha_{SR_k}|^2 + N_0}} \right)$ is the amplification factor. The

instantaneous equivalent end-to-end SNR at the destination can be expressed as

$$\gamma_D = \gamma_{SD} + \sum_{k=1}^N \frac{\gamma_{SR_k} \gamma_{R_kD}}{1 + \gamma_{SR_k} + \gamma_{R_kD}}. \quad (5)$$

Where $\gamma_{SD} = (E_s/N_0) |\alpha_{SD}|^2$, $\gamma_{SR_k} = (E_s/N_0) |\alpha_{SR_k}|^2$ and

$\gamma_{R_kD} = (E_s/N_0) |\beta_{R_kD}|^2$ are the instantaneous SNRs in the S→D,

S→R_k and R_k→D links, respectively. Here, N_0 is the noise variance. In order to simplify the analysis an upper bound [26] for the second term of the instantaneous SNR in (5) corresponding to k^{th} relay can be given as

$$\gamma_k = \min(\gamma_{SR_k}, \gamma_{R_kD}) \geq \frac{\gamma_{SR_k} \gamma_{R_kD}}{1 + \gamma_{SR_k} + \gamma_{R_kD}}. \quad (6)$$

Finally, Eq. (5) can be simplified to

$$\gamma_D \leq \gamma_{SD} + \sum_{k=1}^N \gamma_k. \quad (7)$$

The MGF of the total SNR (end-to-end SNR) can be used to analyze the performance of the system. The total SNR can be written as $\gamma_{\text{tot}} = \gamma_{SD} + \gamma_b$ where γ_b is instantaneous SNR corresponding to best relay and the MGF of γ_{tot} can be expressed as

$$M_{\gamma_{\text{tot}}}(s) = M_{\gamma_{SD}}(s) M_{\gamma_b}(s). \quad (8)$$

Where $M_{\gamma_{SD}}(s)$ and $M_{\gamma_b}(s)$ are the MGF of γ_{SD} and γ_b , respectively.

III. SYMBOL ERROR RATE ANALYSIS

Based on the moment generating function approach, the exact SER expression for MPSK is given by

$$P_s = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\text{tot}} \left(s = -\frac{\sin^2(\pi/M)}{\sin^2\theta} \right) d\theta. \quad (9)$$

Where M is the modulation order. In next subsection, the MGF expressions of γ_b and γ_{SD} are derived in details.

A. MGF of the SNR in S→R_k→D links

MGF expression can be calculated by taking the Laplace transform of the PDF expression. Assuming all links are *double* Rayleigh fading channel, the PDF and CDF of γ_a

($a \in \{SR_k, R_kD\}$) are given by [17], [18],

$$f_{\gamma_a}(\gamma) = \frac{2}{\bar{\gamma}_a} K_0 \left(2\sqrt{\frac{\gamma}{\bar{\gamma}_a}} \right) \quad (10)$$

$$F_{\gamma_a}(\gamma) = 1 - 2\sqrt{\frac{\gamma}{\bar{\gamma}_a}} K_1 \left(2\sqrt{\frac{\gamma}{\bar{\gamma}_a}} \right) \quad (11)$$

where $\bar{\gamma}_a = E_s/N_0$, $K_0(\cdot)$ and $K_1(\cdot)$ are the modified Bessel functions of the second kind of zero order and first order, respectively [25].

Assuming γ_b is the instantaneous SNR of the best relay, the relay selection algorithm can be expressed as [10]:

$$\gamma_b = \max_{k \in R} \{\gamma_k\} = \max_{k \in R} \left\{ \min(\gamma_{SR_k}, \gamma_{R_kD}) \right\}. \quad (12)$$

Where γ_{SR_k} and γ_{R_kD} are independent. Thus, the CDF of γ_b

can be written by using order statistics as [26]

$$F_{\gamma_b}(\gamma) = \left[F_{\gamma_k}(\gamma) \right]^N. \quad (13)$$

Here N is the number of relays. Calculating the derivative of the CDF in (13) with respect to γ , the PDF expression can be found as

$$f_{\gamma_b}(\gamma) = N \frac{2}{\bar{\gamma}_k} K_0 \left(2\sqrt{\frac{\gamma}{\bar{\gamma}_k}} \right) \left[1 - 2\sqrt{\frac{\gamma}{\bar{\gamma}_k}} K_1 \left(2\sqrt{\frac{\gamma}{\bar{\gamma}_k}} \right) \right]^{N-1} \quad (14)$$

, where $\bar{\gamma}_k = \bar{\gamma}_{SR_k} \bar{\gamma}_{R_kD} / (\bar{\gamma}_{SR_k} + \bar{\gamma}_{R_kD})$.

The MGF of γ_b can be calculated by using the Laplace transform as

$$M_{\gamma_b}(s) = \int_0^{\infty} \exp(-s\gamma) f_{\gamma_b}(\gamma) d\gamma. \quad (15)$$

With the help of [25, Eq. 6.614.4] and [25, Eq. 7.813.1], the closed form expression of MGF when N = 2 can be calculated as,

$$M_{\gamma_b}(s) = \frac{2}{\sqrt{s\bar{\gamma}_k}} \exp\left(\frac{1}{2s\bar{\gamma}_k}\right) W_{\frac{1}{2},0}\left(\frac{1}{s\bar{\gamma}_k}\right) - \frac{4\sqrt{\pi}}{(s\bar{\gamma}_k)^{3/2}} G_{2,3}^{3,1}\left(\frac{4}{s\bar{\gamma}_k} \left| \begin{matrix} -\frac{1}{2}, 0 \\ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \end{matrix} \right.\right) \quad (16)$$

where $W_{\dots}(\cdot)$ is the Whittaker function and $G_{\dots}(\cdot)$ is the MeijerG function [25]. For other N values, Expression (15) can be evaluated easily by using common mathematical software packages such as MATHEMATICA or MAPLE.

B. MGF of the SNR in S→D link

The PDF of end-to-end SNR in S→D link (γ_{SD}) which is modelled as *double* Rayleigh fading channel can be written as,

$$f_{\gamma_{SD}}(\gamma) = \frac{2}{\bar{\gamma}_{SD}} K_0\left(2\sqrt{\frac{\gamma}{\bar{\gamma}_{SD}}}\right) \quad (17)$$

where $\bar{\gamma}_{SD} = E_s/N_0$. The CDF expression can be written by taking the integral of PDF in (17) as,

$$F_{\gamma_{SD}}(\gamma) = 1 - 2\sqrt{\frac{\gamma}{\bar{\gamma}_{SD}}} K_1\left(2\sqrt{\frac{\gamma}{\bar{\gamma}_{SD}}}\right) \quad (18)$$

By using the [25, Eq. 6.614.4], the MGF of γ_{SD} can be calculated as,

$$M_{\gamma_{SD}}(s) = \frac{1}{\sqrt{s\bar{\gamma}_{SD}}} \exp\left(\frac{1}{2s\bar{\gamma}_{SD}}\right) W_{\frac{1}{2},0}\left(\frac{1}{s\bar{\gamma}_{SD}}\right) \quad (19)$$

III. SIMULATION RESULTS

In this section, Monte-Carlo simulations are presented to confirm the derived numerical results. We consider a cooperative AF relaying system with relay selection for MPSK modulation. All the links between S→D, S→R_k and R_k→D are assumed as *double* Rayleigh fading channel. For comparison, a cooperative diversity system with relay selection in which all links have Rayleigh fading distribution is provided as benchmark. The simulation results are presented for BPSK and QPSK modulations and the number of relays is assumed as N=2, 3, 4. We also assume that $\bar{\gamma}_{SD} = \bar{\gamma}_0 = E_s/N_0$. Thus $\bar{\gamma}_k = \bar{\gamma}_0/2$. The SNR comparisons are done at a BER of 10⁻⁴.

In Fig.2, the SER performance of considered system is presented for BPSK modulation when the number of relay is equal to 2, N=2. The obtained results clearly show that the SER performance of cooperative system with relay selection is better when all links have Rayleigh distribution. The SNR gains provided by the system with Rayleigh distribution to the system with *double* Rayleigh distribution is about 5.8 dB.

In Fig.3, we present the average SER performance of the system for QPSK modulation with the number of relays is N=2. Comparing with Fig.2 it can be seen that the performance of considered system with BPSK modulation is better than the one obtained with QPSK modulation. The gain is about 4.5 dB. And also Fig. 3 shows that, system with Rayleigh distribution have better performance than the system with double Rayleigh distribution.

In Fig.4, the SER performance of the system is obtained with the different number of relays (N=2,3,4) for BPSK modulation when all links have *double* Rayleigh distribution. As expected,

there is a direct proportion between the number of relays and the average SER performance. The performance of considered system can be increased with the increment of number of the relays.

The numerical results show the lower bound for the simulation results. We can also observe that the derived lower bound results for the average SER are tight enough to the simulation results.

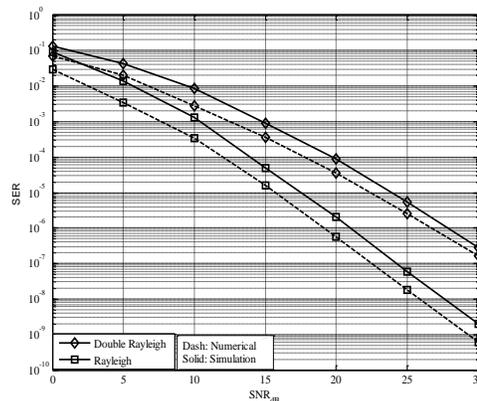


Fig.2. SER performance of AF relaying system with relay selection for BPSK modulation when N=2.

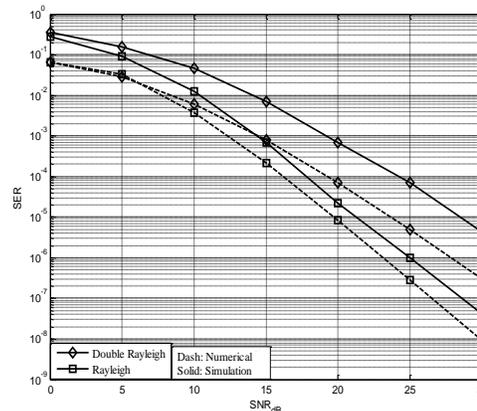


Fig.3. SER performance of AF relaying system with relay selection for QPSK modulation when N=2.

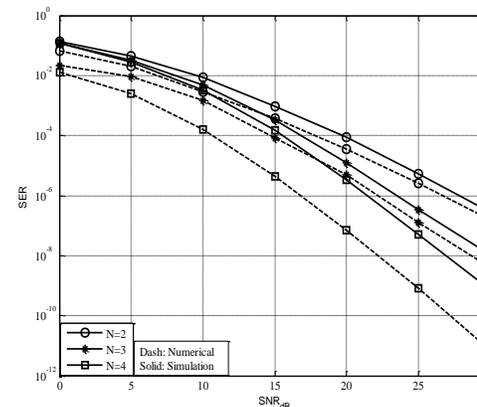


Fig.4. SER performance of AF relaying system over double Rayleigh fading channels for BPSK modulation.

IV. CONCLUSION

In this paper, the performance analysis of an AF based cooperative diversity system with relay selection is discussed when all links between $S \rightarrow R_k$, $R_k \rightarrow D$ and $S \rightarrow D$ are double Rayleigh fading channel. We derive the analytical expressions for the PDF, the CDF and the MGF of the end-to-end SNR of the considered system. The average symbol error rate based on MGF expression is derived. The simulation results show that the performance of cooperative communication system can be increased with relay selection.

V. ACKNOWLEDGEMENTS

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REFERENCES

- [1]. Laneman JN, Tse D and Wornell G (2004). Cooperative diversity in wireless networks: efficient protocols and outage behavior. *IEEE Trans. Inf. Theory*, 50:3062-3080.
- [2]. Nabar RU, Bölcskei H and Kneubühler FW (2004). Fading relay channels: Performance limits and space-time signal design," *IEEE J. Sel. Areas Commun.*, 22:1099-1109.
- [3]. Suraweera HA, Louie RHY, Li YH, Karagiannidis GK and Vucetic B (2009). Two hop amplify-and-forward transmission in mixed Rayleigh and Rician fading channels. *IEEE Commun. Lett.*, 13: 227-229.
- [4]. Lei X, Fan P, Fan L, Wong KK, Larsson EG and Duong TQ (2011). Closed-form performance analysis of multihop amplify-and-forward transmission in Nakagami-m fading channels. *IEEE Signal Process. Lett.*, 18:454-457.
- [5]. Sendonaris A, Erkip E, and Aazhang B (2003). User cooperation diversity Part I: System description. *IEEE Trans. Commun.*, 51:1927-1938.
- [6]. Sendonaris A, Erkip E, and Aazhang B (2003). User cooperation diversity Part II: Implementation aspects and performance analysis. *IEEE Trans. Commun.*, 51:1939-1948.
- [7]. Nosratinia A, and Hunter TE (2006). Diversity through coded cooperation. *IEEE Trans. Wireless Commun.*, 5:283-289.
- [8]. Zhao Y, Adve R, and Lim TJ (2006). Symbol error rate of selection amplify-and-forward relay systems. *IEEE Commun. Lett.*, 10:757-759.
- [9]. Zhang X and Gong Y. Adaptive power allocation in two-way amplify-and-forward relay networks. In *Proc. IEEE ICC*, Dresden, Germany, 2009, pp. 1-5, 2009.
- [10]. Torabi M, Haccoun D and Ajib W (2010). Performance analysis of cooperative diversity with relay selection over non-identically distributed links. *IET Commun.*, 4:596-605.
- [11]. Beres E, Adve R (2008). Selection cooperation in multi-source cooperative networks. *IEEE Trans Wireless Commun* 7:118-127.
- [12]. Akki AS and Haber F (1986). A statistical model of mobile-to-mobile land communication channel. *IEEE Trans. Veh. Technol.*, 35:2-7.
- [13]. Kovacs IZ. Radio channel characterization for private mobile radio systems: Mobile-to-mobile radio link investigations. Ph.D. Thesis, Aalborg University, Sept. 2002.
- [14]. Andersen JB. Statistical distributions in mobile communications using multiple scattering," *URSI General, Assembly Proc.*, Session CBF, Aug. 2002.
- [15]. Salo J, El-Sallabi H and Vainikainen P (2006). Statistical analysis of the multiple scattering radio channel. *IEEE Trans. Antennas Propag.*, 54:3114-3124.
- [16]. Karagiannidis GK, Sagias NC and Mathiopoulos PT (2007). N*Nakagami: A novel stochastic model for cascaded fading channels. *IEEE Trans. Commun.*, 55:1453-1458.
- [17]. Uysal M (2005). Maximum achievable diversity order for cascaded Rayleigh fading channels. *IET Electronics Letter*, 41:1289-1290.
- [18]. Ilhan H, Uysal M and Altunbas I (2009). Cooperative diversity for inter-vehicular communication: Performance analysis and optimization. *IEEE Trans. Veh. Technol.*, 58(7):3301-3310.
- [19]. Ilhan H, Altunbas I and Uysal M (2010). Novel distributed space-time trellis codes for relay systems over cascaded Rayleigh fading. *IEEE Commun. Lett.*, 14:1140-1142.
- [20]. Seyfi M, Muhaidat S, Liang J and Uysal M (2011). Relay selection in dual-hop vehicular networks. *IEEE Signal Process. Lett.*, 18:134-137.
- [21]. Ahmed S, Yang LL, Hanzo L. Probability distributions of products of Rayleigh and Nakagami-m variables using Mellin transform. *International conference on communications (ICC)*, Kyoto, 2011.
- [22]. Shankar PM. Performance of N*Nakagami cascaded fading channels in dual selection combining diversity. *7th international IEEE wireless communication mobile computer conference (IWCMC)*, Istanbul, 2011.
- [23]. Zheng Z, Wei L, Hamalainen J. Novel approximations to the statistics of general cascaded Nakagami-m channels and their applications in performance analysis. *IEEE international conference on communications (ICC)*, Budapest, 2013.
- [24]. Ilhan H, Akin AI. Performance analysis of AF relaying cooperative systems with relay selection over double Rayleigh fading channels. *6th international conference on signal processing and communication systems (ICSPCS)*, Gold Coast, 2012
- [25]. Gradshteyn IS and Ryzhik MI (2007). Table of integrals, series and products. Academic Press, 7nd ed.
- [26]. Anghel P and Kaveh M (2004). Exact symbol error probability of a cooperative network in a Rayleigh-fading environment fading channels. *IEEE Trans. Wireless Commun.*, 3:1416-1421.