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N-JOBS AND M-MACHINES FLOWSHOP SCHEDULING TO MINIMIZE THE RENTAL COST

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Abstract— **This paper considers a specially structured n-jobs and m-machines flow shop scheduling in which the processing times are associated with probabilities. The objective of this problem focuses on minimization of the total rental cost of the machines under a specified rental policy. In recent years, researchers have suggested many heuristic procedures to minimize the makespan, but in many cases the minimization of makespan does not give the minimum rental cost of the machines. An algorithm is proposed to solve the problem.**

Index Terms— Flowshop scheduling, utilization time, idle time, makespan, rental cost.

I. INTRODUCTION

Flowshop scheduling problem is one of the most studied problem in the scheduling literature. The objective of this problem generally focuses to minimize the rental cost of the machines. Besides this, total flow time, idle time are also considered. First research on flowshop scheduling problem has been done by Johnson (1954). Johnson developed an exact algorithm for n tasks and two-machines flowshop scheduling problem with objective of makespan. After the Johnson's paper, many exact algorithms and heuristics have been proposed for solving flowshop scheduling problems with different objectives. Ignall and Schrage (1965), =Lominicki (1965), Ashour (1970), Mcmahon and Burton (1967), Stafford (1988) have been proposed exact solutions for this problem. Exact algorithms are limited by the problem size to solve, as they become impractical for large size problems. When the flow shop scheduling problem enlarges as including more jobs and machines, it becomes a combinatorial optimization problem. Combinatorial optimization problems are in NP-hard problem class, and approximate optimum solutions are preferred for such problems. Several heuristics for the flowshop scheduling problem have been developed by Palmer (1965), Smith and Dudek (1967), Campbell et al. (1970), Gupta (1971), Nawaz et al. (1983), Rajendran and Ziegler (1997), Lui and Reeves (2001), Framinan and Leisten (2003), Kalczynski and Kamburowski (2007), Li et al. (2009), Rad et al. (2009). The aim of this paper is to develop a heuristic algorithm to minimize the total rental cost of the machines under a specified rental policy having n-jobs and m-machines in which the processing times are associated with probabilities.

II. PRACTICAL SITUATION

There are many practical situations in real life in which we have to finish some assignment using various machines and it is not feasible to purchase machines, e.g. In the field of Medical, production units, construction of buildings etc. In these situations we take these machines on rent in order to complete the assignment.

III. NOTATIONS

S : Sequence of jobs

M_j : Machine j, $j = 1, 2, ..., m$

t_{ij} : Processing time of i^{th} job on machine M_j

p_{ij}: Probability associated to the i^{th} job on machine M_j

A_{ij}: Expected processing time of i^{th} job on machine M_j

$t_{ij}(S)$: Completion time of the i^{th} job of sequence S on machine M_j

$I_{ij}(S)$: Idle time of machine M_j for job i in the sequence S

$U_j(S)$: Utilization time of machine M_j for the sequence S

C_j : Rental cost of machine M_j

$R(S)$: Total rental cost for the sequence S of all the machines

$CT(S)$: Total completion time of the jobs for the sequence S

Completion time of the i^{th} job on machine M_j is denoted by t_{ij} i.e.

$T_{ij} = \max(T_{i-1j}, T_{ij-1}) + A_{ij}$, where $A_{ij} = t_{ij} \times p_{ij}$ for $j \geq 2$.

The machines will be taken on rent as and when they are required and are returned as and when they are not in use i.e. the machine M_1 will be taken on rent in the starting of the processing of the first job of sequence S, machine M_2 will be taken on rent at time when first job of sequence S is completed on machine M_1 and the machine M_3 will be taken on rent when the first job of sequence S is completed on machine M_2 and so on.

IV. PROBLEM FORMULATION

Consider we have *n*-jobs to be processed on *m*-machines M_j ($j = 1, 2, ..., m$) under the specified rental policy. Let t_{ij} be processing time of i^{th} job on machine M_j with probabilities p_{ij}. Let A_{ij} be the expected processing time of i^{th} job on machine M_j. Our aim is to find the sequence S of the jobs which minimize the total rental cost of the machines. The minimum value of total rental cost for the sequence S is obtained as:

$$R(S) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S) \times C_2 + ... + U_m(S) \times C_m$$

Table 1: The mathematical model of the problem in matrix form

Job (i)	Machine M_1		Machine M_2		....	Machine M_m	
	t_{i1}	p_{i1}	t_{i2}	p_{i2}	...	t_{im}	p_{im}
1	t_{11}	p_{11}	t_{12}	p_{12}	...	t_{1m}	p_{1m}
2	t_{21}	p_{21}	t_{22}	p_{22}	...	t_{2m}	p_{2m}
.	.	.	.	.	...	.	.
.	.	.	.	.	...	.	.
.	.	.	.	.	...	.	.
n	t_{n1}	p_{n1}	t_{n2}	p_{n2}	...	t_{nm}	p_{nm}

V. ALGORITHM

Step 1: Calculate the expected processing times as:
$A_{ij} = t_{ij} \times p_{ij}$

Step 2: Find the sum of the expected processing times for each machine.

Step 3: Select the machine for which $Min\left\{\sum_{i=1}^{n} A_{ij}\right\}$, (for all *j*) occurs.

Step 4: Obtain the sequence S by using the expected processing times in descending order of the machine selected in step-3.

Step 5: Find R(S) to obtain the minimum rental cost.

VI. NUMERICAL ILLUSTRATION

Consider 5 jobs, 3 machine flow shop problem with processing time associated with their respective probabilities as given in the following table. The rental cost per unit time for machines M_1, M_2 and M_3 are 4 units, 6 units and 8 units respectively, under the rental policy P. Our objective is to obtain an optimal sequence of the jobs to minimize the rental cost of the machines.

Job (i)	Machine M_1		Machine M_2		Machine M_3	
	t_{i1}	p_{i1}	t_{i2}	p_{i2}	t_{i3}	p_{i3}
1	100	0.1	30	0.2	50	0.2
2	50	0.2	20	0.4	80	0.1
3	40	0.3	40	0.1	50	0.2
4	48	0.2	20	0.2	40	0.3
5	40	0.2	60	0.1	40	0.2

Solution: The expected processing times A_{i1}, A_{i2} and A_{i3} for machines M_1, M_2 and M_3 are given in the following table

Job (i)	Machine M_1 A_{i1}	Machine M_2 A_{i2}	Machine M_3 A_{i3}
1	10	6	10
2	10	8	8
3	12	4	10
4	9.6	4	12
5	8	6	8

Using step-2, we have $\sum_{i=1}^{n} A_{i1}$=49.6, $\sum_{i=1}^{n} A_{i2}$=28 and $\sum_{i=1}^{n} A_{i3}$=48, and $Min\{49.6, 28, 48\} = 28$ occurs for machine M_2 so, we have

Job (i)	1	2	3	4	5
A_{i3}	6	8	4	4	6

Now, we have the sequence S by using the expected processing times in the above table in descending order as:

Sequence $S = \{2,1,5,\ 3,\ 4\}$

Job (i)	Machine M_1 In - Out	Machine M_2 In - Out	Machine M_3 In - Out
2	0 – 10	10- 18	18 - 26
1	10 - 20	20 - 26	26 - 36
5	20 - 28	28 - 34	36 - 44
3	28 - 40	40 - 44	44 - 54
4	40– 49.6	49.6 – 53.6	53.6 – 65.6

The total completion time for the sequence $S = CT\ (S) = 65.6$ units, utilization time of machine $M_2 = U_2\,(S) = 43.6$ units and the utilization time of machine $M_3 = U_3(S) = 47.6$ units.

Hence, the minimum rental cost

$$R(S) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S) \times C_2 + U_3(S) \times C_3$$

$$= 49.6 \times 4 + 43.6 \times 6 + 23.8 \times 8 = 650.4 \text{ units}$$

CONCLUSION

We have developed a heuristic procedure for specially structured n-jobs and m-machines flow shop scheduling in which the processing times are associated with probabilities. We obtained a schedule with a set of n-jobs to minimize the total rental cost of the machines under a specified rental policy. This method is very easy to understand and to apply. It will also help managers in the scheduling related issues by aiding them in the decision making process.

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