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DOPPLER ESTIMATION OF SINUSOIDAL SIGNAL USING A SPARSE REPRESENTATION

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***Abstract*— In this paper, a new technique that estimates velocity using sparse representation is suggested. The CW radar is effective for velocity estimation because it uses the Doppler frequency shift. Traditional Doppler shifts are calculated using a Fast Fourier Transform (FFT), which is needed to obtain a sufficient number of received signals for high-resolution. The maximum frequency resolution is proportional to the number of received signals, and accurate velocity estimation requires a long integration time. When a moving target has a velocity of 30km/h and a carrier frequency is 10GHz, received signal has a Doppler frequency of 55Hz and noise. The length of the dictionary and the receive signal can be reduced to 0.2% by multiplying a normalized random matrix. If the received Doppler frequency is close to a frequency in the dictionary and unique which means that the sparsity is one, then the least square error is small between the received signal and the dictionary. A minimum least square error of the frequency is represented by the best-fit solution instead of L1 minimization because the sparsity of the solution is well known. The solution has a dependency with noise but the proposed method is much faster and more robust. This study proves that in Doppler estimation, a proposed method improves frequency accuracy even though the received data reduce to 0.2% of samples.**

***Index Terms*—Sinusoidal Signal, Doppler Estimation, Sparse Representation**

I. INTRODUCTION

In spite of noise limitation, continuous-wave (CW) radar is widely used in near range. The CW radar is effective for velocity estimation because it uses the Doppler shift. Traditional Doppler frequency is calculated using a Fast Fourier Transform (FFT), which is needed to obtain a sufficient number of received signals for high-resolution. The maximum frequency resolution is proportional to the number of received signals, and accurate velocity estimation requires a long integration time. In this paper, a new technique that estimates velocity using compressed sensing is suggested.

II. CW RADAR AND DOPPLER RADAR

A. CW Radar

CW radar has several narrow Doppler filter banks and calculates Doppler frequency using FFT.

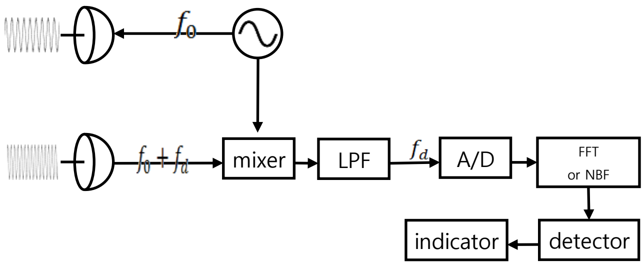

Fig. 1. **Classical CW radar block diagram**

If the Doppler radar estimates a Doppler frequency using FFT, frequency resolution is determined by dwell time.

$$\Delta f = \frac{1}{T_{Dwell}}$$

where B is the maximum resolvable frequency and NFFT is the length of the data. The maximum resolvable frequency depends on the number of signals and dwell time.

B. Sinusoidal Signal Model

It is assumed that the received signal has an arbitrary phase delay, and the target is an object. Therefore, the signal has only one frequency, and the signal model can be expanded above the equation. The received signal can be represented by a linear combination of sine and cosine functions with the constants c1 and c2.

$$s(t) = \cos(2\pi f_d t + \phi) = \cos(\phi)\, cos(2\pi f_d t) - \sin(\phi)\, sin(2\pi f_d t)$$
$$= C_1 cos(2\pi f_d t) + C_2 sin(2\pi f_d t)$$

where f_d is the Doppler frequency and ϕ is an arbitrary phase delay.

C. Frequency Vector

It is assumed that a linear combination of restricted numbers of M frequencies represent the received signal, and ω_i is the frequency vector.

$$\omega_i = \begin{bmatrix}\cos(2\pi\omega_i t)\\ \sin(2\pi\omega_i t)\end{bmatrix}^T$$

where t= [0, t_s, $2t\_s$, ⋯, Nt_s], t_s is the sampling interval and i=1,2,⋯,M.

III. METHODS

A. Method

The dictionary matrix, A, consists of several frequencies of both sine and cosine functions.

$$A = [\omega_1, \omega_2, \omega_3, \ \cdots, \omega_M] \in \mathbb{R}^{N\times 2M}$$

Coefficients of linear combinations are expressed below.

$$x = [x_1, x_2, x_3, \cdots, x_{2M}]^T \in \mathbb{R}^{2M}$$

The received signal, y, is expressed below.

$$y = Ax \in \mathbb{R}^N$$

Due to the solution being sparse, the dimension of matrix, A, can be reduced by multiplying a normalized random matrix, R, which is a character of compressed sensing.

$$y' = Ry = RAx \in \mathbb{R}^P, \qquad R \in \mathbb{R}^{PxN}$$

B. Solution

When a moving target is ideal, received signal has a Doppler frequency and noise. The solution should be in the form shown below.

$$\hat{x}_k = [0,0,\cdots,0,\hat{x}_{2k-1},\hat{x}_{2k},0,\cdots,0\,]^T, \quad k = 1,2,\cdots,M$$

To solve this problem, the L2 minimization of each frequency is used. Residuals of the L2 minimum, r_k, are square errors between the received signal and a linear combination of the frequency vector.

$$r_k = ||y - A\hat{x}_k||_2$$

If the received Doppler frequency is close to the frequency of the dictionary, then the least square error is small, ant the solution frequency has a minimum residual.

$$r_{\hat{k}} = argmin\, r_k$$

The target velocity can be easily estimated using the below equation.

$$v = c\frac{f_d}{2f_0} = c\frac{2\pi\omega_{\hat{k}}}{2f_0}, \qquad \hat{k} = 1,2,\ \cdots, M$$

Figure 2 is represented by a block diagram of the suggested method—the constrained least square method.

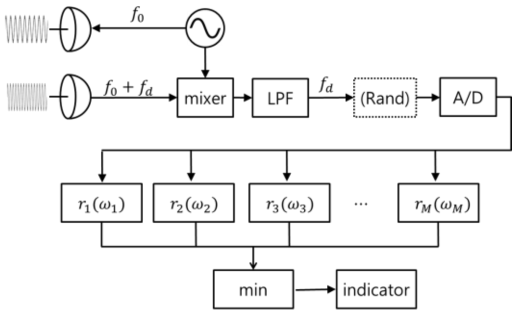

Fig. 2. **Constrained least square method for CW radar block diagram.**

IV. RESULTS

Parameters	Case 1	Case 2	Case 3	Case 4
Dwell Time	4ms			
# of Samples	6441		128	
Sampling Rate	16.1MHz		32KHz	
Carrier Frequency	10GHz			
Target Velocity(Km/h)	30			
Doppler Frequency(Hz)	555.56			
Maximum Resolvable f, B(Hz)	8.05M	8.05M	160k	160k
Dictionary (km/h)	1:0.1:80			
SNR	0dB	-8dB	0dB	-8dB

The simulations use the above parameters with uniformly pseudo-random noise.

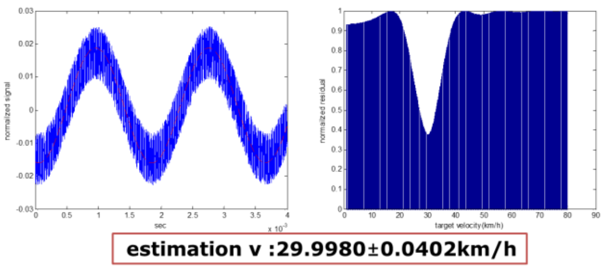

Fig. 3. **Case 1 results (a) received signal (b) coefficients of dictionary**

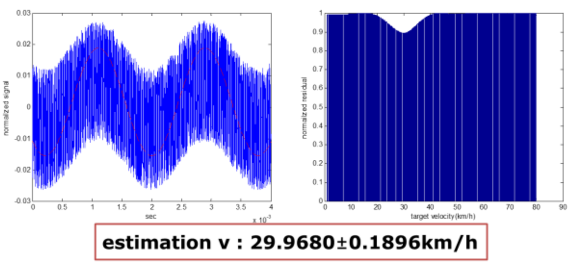

Fig. 4. **Case 2 results (a) received signal (b) coefficients of dictionary**

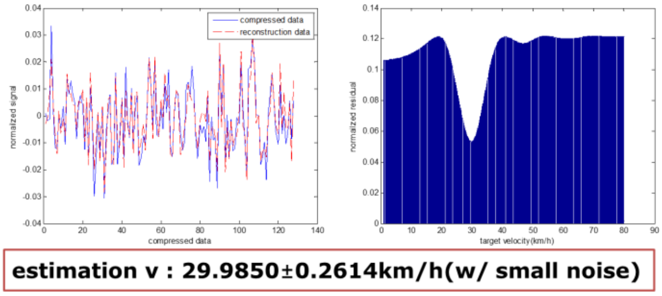

Fig. 5. **Case 3 results (a) received signal (b) coefficients of dictionary**

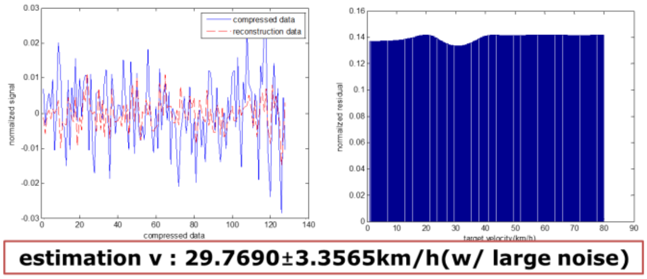

Fig. 6. **Case 4 results (a) received signal (b) coefficients of dictionary**

If the solution has a property of sparse, then it can be solved using compressed sensing. In this problem, the sparsity of the solution is too well known, and the L1 minimization is not necessary. The length of the dictionary can be reduced to 128 by multiplying a normalized random matrix.

V. CONCLUSIONS

This study has demonstrated that the constrained least square method decreases the data-sampling rate and improves frequency accuracy. The solution frequency has a minimum least square error and is represented by the best-fit solution in time domain without FFT. This proposed method improves frequency resolution, and is much faster and robust.

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