International Journal of Technical Research and Applications e-ISSN: 2320-8163,
www.ijtra.com Special Issue 23 (July 2015), PP.29-32

ROLL AUTOPILOT DESIGN USING VARIABLE STRUCTURE CONTROL BASED ON NEW REACHING LAW

Daqin Luo[1], Yongshan Liu[2]
School of Aerospace Engineering
Beijing Institute of Technology
China, Beijing
[1]luodaqinde@163.com
[2]liuysh@bit.edu.cn

***Abstract*— The variable structure roll autopilot is designed with reaching law approach. Aiming at solving the chattering problem caused by sign function, the saturation function is introduced. In order to improve the system, a new reaching law is proposed on the basis of the conventional exponential reaching law. Furthermore, to prove the efficiency of the variable structure control autopilot, a classical autopilot is also designed. Simulation results indicate the efficiency of variable structure control autopilot, and the new reaching law improves the system more..**

***Index Terms*— missile, roll autopilot, classical control, sliding mode variable structure control, chattering**

I. INTRODUCTION

To control airframe with feedback information of inertial device, and produce both good dynamic characteristics and steady-state characteristics, an autopilot should be designed [6]. Autopilot and its applications have been extensively studied over several decades in the past. The two-loop acceleration autopilot based on classical control theory has been widely used because it is both easy to implement and has high reliability. Aside from these advantages, the classical control also has drawback that airframe kinetic parameters various a lot throughout the flight envelope due to the affect of many factors such as flight speed, flight height, random disturbance, which leads to model uncertainties. So it is difficult to meet the requirement of robustness with traditional classical feedback control. Aiming at the problem, modern control theory is used in this paper.

Sliding mode variable structure control (SMVSC) respond quickly and is insensitive to external disturbance and parameter variation as a kind of modern control approach, which has attracted more and more attention. The SMVSC is a special kind of nonlinear control strategy characterized by a discontinuous control that changes the system structure with time. The core of the approach is to design a sliding surface and control law [1]. The control law is designed such that the system states are driven to the switching surface (SS), and thereafter remain in the vicinity of the SS along with the corresponding state trajectory [2].

There are three kinds of conventional control scheme, constant switching control, proportional switching control and function switching control. The first two schemes can only guarantee the existence and reach ability of sliding mode, but don’t restrict the time and trajectory reaching to the switching surface of the states, which means the control effect can’t be guaranteed. The function switching scheme can utilize the reaching law to adjust the velocity and trajectory of states reaching to the switching surface such that dynamic quality of sliding motion can be improved [4].

Conventional reaching law includes constant reaching law, exponential reaching law and so on. The engineering experience shows that the exponential has better effectiveness, so this paper only study the exponential reaching law aiming at conventional reaching law. On the other hand, nowadays, the missiles require higher precision, and in order to meet the higher requirement, a new reaching law is proposed in this paper.

- Variable structure roll autopilot design

As shown in Fig.1, variable structure control roll autopilot consists of variable structure controller (VSC), actuator, plant, rate gyro and angle gyro as shown in Fig.1. Where γ_c is autopilot input, u is control law, $\dot{\gamma}$ is roll rate and γ is roll angle. Note Fig.1, the VSC is the part to be designed, which consists of two parts: the sliding surface design and the control law design, and the design approach will be stated later.

○ *Mathematical model*

The missile transfer function of roll channel is derived here with the aileron angle δ_x as input and the roll angle γ as output:

$$\frac{\gamma(s)}{\delta_x(s)} = \frac{k_r}{s(T_r s+1)} \tag{1}$$

The Eq.1 can be rewritten in the phase variable state-space model as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = A\begin{bmatrix}x_1\\ x_2\end{bmatrix} + Bu$$

$$y = C\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad (2)$$

Where $A = \begin{bmatrix}0 & 1\\ 0 & -\frac{1}{T_r}\end{bmatrix}$, $B = \begin{bmatrix}0\\ \frac{k_r}{T_r}\end{bmatrix}$, $C = [1 \quad 0]$, k_γ is the aerodynamic gain and T_γ is time constant. The system state variable are $x_1 = \gamma$, $x_2 = \dot{x}_1 = \dot{\gamma}$, the output is $y = x_1 = \gamma$, the control $u = \delta_x$. The autopilot is designed in the feature point of 1.6 Ma, in this point, $k_\gamma = -25.61$, $T_\gamma = 5.83$.

- *Sliding surface design*

The sliding surface is chosen in form of

$$\sigma(x) = c_s\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad (3)$$

Where c_s is a 1×2 dimension matrix which will affect the dynamic quality, and considering both rapidity and stability, c_s will be chosen as $[8 \quad 1]$.

The motion of variable structure control system is composed of normal motion mode and sliding motion mode. The normal mode is a reaching mode where the system beginning from arbitrary initial sate will be attracted towards the SS $\sigma(x) = 0$. In the sliding mode, the system states slide along the SS $\sigma(x) = 0$, at last arrive $x_1 = 0, x_2 = 0$. Once the sliding surface has been chosen, the corresponding controller should be designed to make $\sigma(x) = 0$ be an attractive surface [2].

- *Control law design*

Choosing the Lyapunov candidate function as

$$W = \frac{1}{2}\sigma(x)^T\sigma(x) \qquad (4)$$

Differentiation of Eq.4 is $\dot{W} = \frac{1}{2}\sigma(x)^T\dot{\sigma}(x)$, if $\dot{W} < 0$, then the existence, reach ability, stability of the sliding mode will be met.

Considering $\sigma(x) = c_s x$, differentiation of $\sigma(x)$ is the reaching law, and it can be expressed as follows:

$$\dot{\sigma}(x) = c_s\dot{x} \qquad (5)$$

Substituting Eq.5 into Eq.2 yields the control law as follows:

$$u = (c_s B)^{-1}(-c_s Ax + \dot{\sigma}(x)) \qquad (6)$$

The Eq.6 shows that u is dominated by reaching law $\dot{\sigma}(x)$, which means that different reaching laws will lead to the variation of system response.

1).Control law design based on exponential reaching law

The form of exponential reaching law is

$$\dot{\sigma}(x) = c_s\dot{x} = -\varepsilon sgn(\sigma(x)) - k\sigma(x) \qquad (7)$$

It is obvious that Eq.7 meet the condition $\dot{W} < 0$, substituting Eq.7 into Eq.6 yields

$$u = (c_s B)^{-1}(-c_s Ax - \varepsilon\,\mathrm{sgn}(\sigma(x)) - k\sigma(x)) \qquad (8)$$

Note that the sig function $\mathrm{sgn}(\sigma(x))$ is a discontinuous function which can cause chattering phenomenon. As a practical matter, chattering is undesirable because it is too fast to respond. The discontinuity can be dealt with by adopting thin boundary layer around the switching surface [2]. So we replace $\mathrm{sgn}(\sigma(x))$ with continuous saturation function $sat(\sigma(x))$, where $sat(\sigma(x)) = \frac{\sigma(x)}{|\sigma(x) + \Delta|}$, and Δ =0.1. Then the reaching law (7) can be rewritten as

$$\dot{\sigma}(x) = c_s\dot{x} = -\varepsilon sat(\sigma(x)) - k\sigma(x) \qquad (9)$$

Then the control law (8) can be rewritten as follows:

$$u = (c_s B)^{-1}(-c_s Ax - \varepsilon sat(\sigma(x)) - k\sigma(x)) \qquad (10)$$

The effect of saturation function $sat(\sigma(x))$ of decreasing chattering will be demonstrated later in chap.3.

2) .Control law design based on new reaching law

The form of new reaching law is

$$\dot{\sigma}(x) = c_s\dot{x} = -\varepsilon sat(\sigma(x)) - k\sigma(x)^2 \cdot sat(\sigma(x)) \qquad (11)$$

The new reaching law (11) also meets $\dot{W} < 0$, then the corresponding control law can be expressed as follows:

$$u = (c_s B)^{-1}\left(-c_s Ax - \varepsilon sat(\sigma(x)) - k\sigma(x)^2 \cdot sat(\sigma(x))\right)$$

(12)

Note the control law (12), we can get that $k\sigma(x)^2 \cdot sat(\sigma(x))$ dominates the Eq.12 because system states are far from switching surface in the reaching mode. But in the sliding mode, the system states are around of switching surface, then the $\varepsilon sat(\sigma(x))$ part dominate the Eq.12. This means that the new reaching law can speed up the system response, at the same time, it can also guarantee better stability.

- Simulation results

The classical autopilot is also designed in this paper to prove the efficiency of variable structure control autopilot. Simulation studies are carried out for the autopilots mentioned before at initial condition corresponding to roll angle=5° , roll rate =0° /s by using the software MATLAB and SIMULINK.

The step response of classical autopilot, autopilot based on sign function exponential reaching law, autopilot based on saturation function exponential reaching law, and autopilot based on new reaching law are shown in Fig.2, Fig.3, Fig.4 and Fig.5 respectively. The control law parameters of last three autopilots are chosen as $\varepsilon = 0.1, k = 100$.

Comparing the Fig.3(b) with Fig.4(b) and Fig.5(b), the efficiency of decreasing chattering by utilizing saturation function is proved. Considering the setting time and overshoot of system response are the two most important indexes of autopilot design, so the three kinds of autopilot (classical autopilot, autopilot based on saturation function exponential reaching law, autopilot based on new reaching law) are compared according to the setting time and overshoot as Tab.1.

Note the Tab.1, the variable structure autopilot is more efficient than classical autopilot, and the variable structure autopilot based on new reaching law speeds up the system response and decreases the overshoot further.

- Conclusion

In this paper a variable structure control roll autopilot based on exponential reaching law is designed. Aiming at the chattering phenomenon, saturation function is introduced. A new reaching law is proposed in order to improve the performance of system response. The classical autopilot is also designed to be compared with the VSC autopilot. The simulation results indicate that the VSC autopilot is efficiency, and the VSC autopilot based on new reaching law improves the system more.

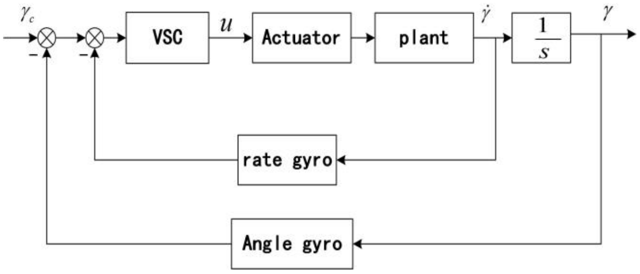
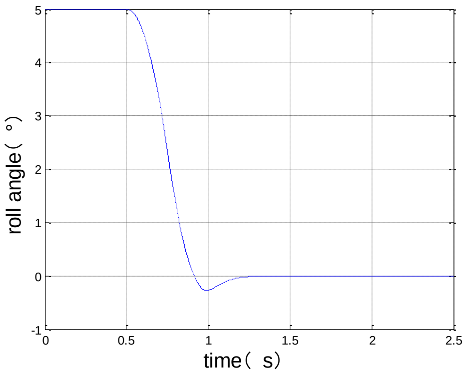
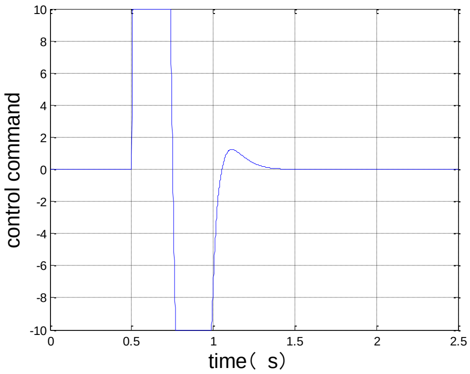


Fig.1 Diagram of variable structure control roll autopilot

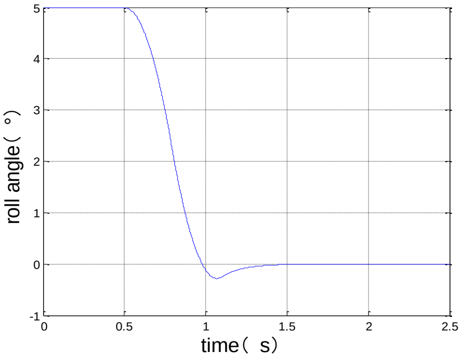


(a) Roll angle

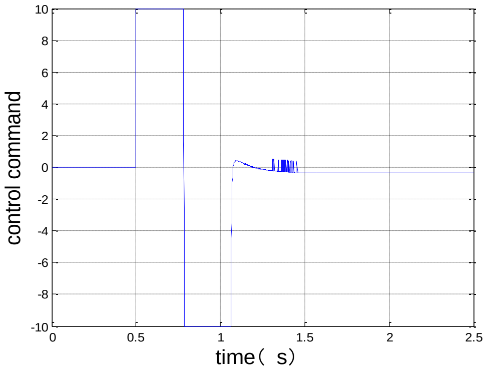


(b) Control command

Fig.2 Response of classical autopilot

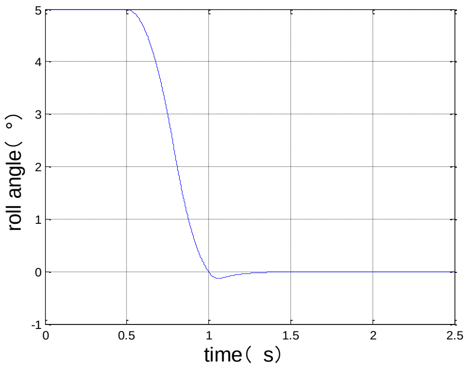


(a) Roll angle

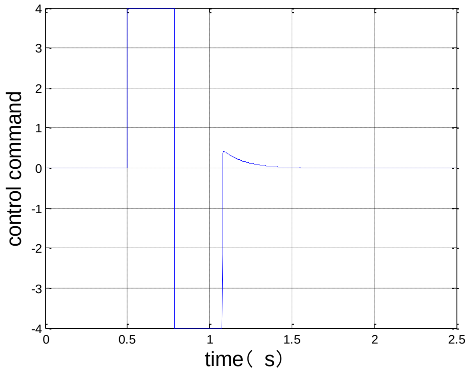


(b) Control command

Fig.3 Response of roll autopilot based on sign function exponential reaching law

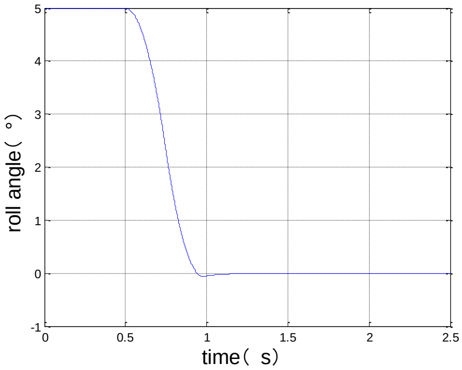


(a) Roll angle

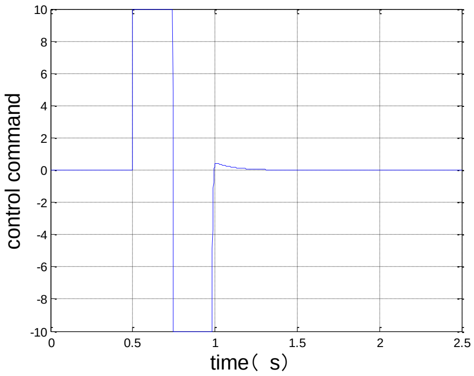


(b) Control command

Fig.4 Response of roll autopilot based on saturation function exponential reaching law



(a) Roll angle



(b) Control command

Fig.5 Response of roll autopilot based on new reaching law

Tab.1 simulation results comparison

index Autopilot	Setting time	Overshoot
Classical autopilot	0.62s	5.5%
Autopilot based on saturation function exponential reaching law	0.45s	2.8%
Autopilot based on new reaching law	0.4s	1.2%

REFERENCES

[1] A.A.Powly and M.Seetharama Bhat. India. “Missile Autopilot Design discrete-time-variable structure controller with sliding sector”. Journal of guidance, control, and dynamics, vol.27,no.4,2004,pp.634-646.

[2] Chae IK Ahn, Youdan Kim, Hyoun Jin Kim. Korea . “adaptive sliding mode controller design for fault tolerant flight control system”. AIAA Guidance, Navigation, and Control Conference and Exhibit, August 2006.

[3] S.E.Talole, A.A.Godbole, and J.P.Kolhe, India,“Robust Roll Autopilot Design for Tactical Missiles”, Journal of guidance, control, and dynamics, Vol. 34, No.1 January-February 2011.

[4] ChenLei, Yongshan Liu, China, “Missile roll autopilot design and simulation”, Journal of Chongqing three gorges university, Vol.28 No.139, 2012

[5] Xingfang Qian, Ruixiong Lin, Missile flight aerodynamic, Beijing: Beijing Institute of Technology Press, 2012

[6] P.Garnell, Guided Weapon Control Systems, Beijing: Beijing Institute of Technology press, 2004

[7] Zhenglin Wang, Shengkai Wang, MATLAB/Simulink and control system simulation, Beijing: Publishing House of Electronics Industry, May 2005.