FORCED VIBRATION OF DAM AND RESERVOIR USING SBFEM

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Abstract: Determination of the hydrodynamic pressure acting on dams during earthquake has been of high interest in design steps of these structures. An efficient method for the analysis of forced vibration of the dam and reservoir in different modes of the dam's motion and deformation has been presented which, could be easily extended to study fluid-structure interaction. The SBFEM (Scaled Boundary Finite Element Method) is utilized to model the far-field sub-domain while the FEM (Finite Element Method) is used to model the near-field sub-domain of the reservoir. Although a simple geometry of boundary conditions has been chosen, all the required necessities to extend the method to a practical case are implemented.

Keywords: Dam-Reservoir Interaction; SBFEM; Hydrodynamic Pressure

I. INTRODUCTION

The SBFEM method has been widely used in the field of dam-reservoir interaction in recent years. Two applications have been considered. First, the method is used to bring the far-field boundary condition to the face of the dam [1]. In the second application, a small zone of the reservoir close to the dam, called the near-field, is meshed using FEM, and the far-field is modeled with SBFEM and finally the coupled FEM-SBFEM is implemented to analyze a semi-infinite reservoir [1]. The second application has the advantage of dealing with irregular geometry of the dam's structure as well as the near-field bottom boundary. The second approach has been utilized in the present study.

In frequency domain, the dam's motion during earthquake may be decomposed into piston, flap, and first mode of deformation all of which are considered as forced vibrating-boundary conditions. Each of these types of deformations as well as their comparisons can be a basis for computing dam-reservoir interaction. Separately, determining each of the contributions of dam's movements, particularly the role of the first-mode dam's deformation, on such dam-reservoir interaction is of considerable significance of this study.

Hydrodynamic pressure distribution acting on dam's surface on the basis of different movements such as piston type, flap type, and first-mode type of dam's deformation with maximum displacement of 0.02 meter at dam's crest has been determined and compared. A comprehensive FORTRAN program has been written to solve the numerical examples presented in this paper, and the results are in a good agreement with analytical solutions.

The analytical computation of hydrodynamic pressure due to the earthquake excitation in the dam-reservoir system has been presented by many researchers. Westergaard [2] for the first time computed the hydrodynamic pressure on upstream face of a rigid dam. In his study, the dam upstream face was vertical, the reservoir bottom was assumed to be parallel to water surface, the fluid was inviscid and incompressible, and the structure was subjected to the horizontal ground motion. Afterwards, Chopra [3] found the hydrodynamic pressure with the same assumptions except for considering both horizontal and vertical ground acceleration as well as compressibility effect of water. Dean et al. determined an analytical solution for hydrodynamic pressure in the flap-type motion of a rigid structure [4].

II. PROBLEM DESCRIPTION

A dam-reservoir system as shown in the figure below is considered in which the reservoir is divided into two sub-domains. For simplicity, the reservoir bottom in the near-field sub-domain is assumed to be horizontal and at interface 1, the dam and fluid interact with each other. The far-field sub-domain is uniform with vertical interface 2, considered as damping radiation boundary condition which is met analytically through SBFEM.

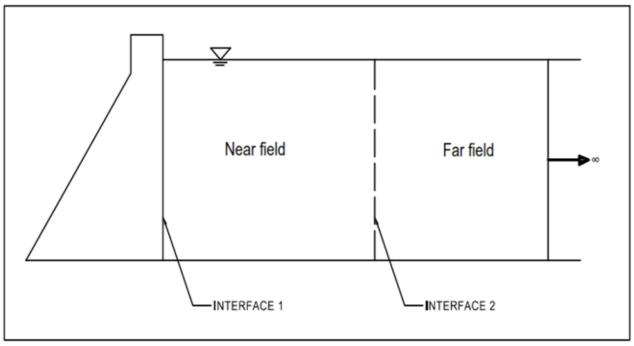

Fig 1.The sketch of dam-reservoir system

A. *Assumptions*

The following assumptions are necessary to compute the hydrodynamic pressure.

1) The effects of waves at free surface of the reservoir are neglected.
2) The fluid is assumed to be inviscid and linearly compressible and the displacements of fluid particles are considered as small.
3) The upstream face of dam is vertical, and the motion of dam- reservoir system is assumed to be two dimensional vertical (2DV).
4) All the points at the base of the dam are assumed to have the same ground motion amplitude.

III. EQUATIONS OF MOTION

A. *FEM Formulation of Near-Field*

The governing equation of fluid in the reservoir, neglecting the body forces is expressed as

$$\nabla^2\varnothing = 1/C^2 \quad \ddot{\varnothing} \qquad (1)$$

Here $\varnothing$ denotes velocity potential and C is wave celerity in fluid.

The velocity and pressure corresponding to the $\varnothing$ are expressed as

$$v = \nabla\varnothing \qquad (2a) \qquad p = -\rho\dot{\varnothing} \qquad (2b)$$

where ρ denotes fluid density. To attain the conventional FEM matrices of this sub-domain, the boundary conditions should be considered as follow

(1) On the interface of dam-reservoir, the normal velocity of fluid particles isspecified:

$$v_n=(\partial\emptyset)/\partial n \quad (3)$$

(2) On the rigid reservoir bottom, we have

$$v_n=(\partial\emptyset)/\partial n=0 \quad (4)$$

(3) The boundary condition of surface water by neglecting the surface waves is considered as

$$\emptyset=0 \quad (5)$$

(4) The damping radiation effect on radiation boundary (interface 2 in figure 1) is automatically implemented in SBFEM.

By weighted residual method and imposing these boundary conditions, the following equation will be obtained.

$$M\ddot{\phi}+K\phi=V_n \quad (6)$$

where

$$V_n=\{■(〖V^'〗_{n1}@V_{n2})\} \quad (7)$$

in which $〖V^'〗_{n1}$ and V_{n2} are normal velocities along interface 1 and interface 2 respectively.

B. SBFEM Formulation of Far-Field

In the following the summary of SBFEM equations in frequency domain is discussed; for more details see the Ref. [5].

The SBFEM formulation on discretized radiation boundary for far-field sub-domain of reservoir is expressed as

$$V_n=S^\infty(\omega) \quad (8)$$

where $S^\infty(\omega)$ is dynamic stiffness matrix of the far-field sub-domain, ω is exciting frequency, and $V_n$is normal velocity matrix of interface 2. It is worth to mention that since water surface and reservoir bottom are parallel in far-field sub-domain, the similarity center in this method is in infinity.

According to the Ref.[5]. $E^1$is equal to zero and therefore we have

$$S^\infty(\omega)=\sqrt{(E^2+〖i\omega C〗^0-\omega^2 M^0)\ 〖(E^0)〗^{-1}}\ E^0 \quad (9)$$

in which E^0, E^2, $M^0$are global coefficient matrices defined in mentioned reference.

Depending on the exciting frequency (ω) and the reservoir's cut-off frequency, the elements of $S^\infty(\omega)$ matrix are real or complex numbers [5].

C. FEM-SBFEM Coupled Formulation

Along interface 2, owing to kinematic continuity, the normal velocities of near-field and far-field sub-domains are the opposite as follow

$$V_{n1}=-〖V^'〗_{n1}=S^\infty(\omega)\phi \quad (10)$$

By substituting Equation (10) into Equation (6) we have

$$M\ddot{\phi}+(K+S^\infty(\omega))\phi=\{■(0@V_{n2})\} \quad (11)$$

where M and K are mass and stiffness matrices of near-field sub-domain.

IV. NUMERICAL EXAMPLES

To avoid the complexity of structure's geometry, the dam is modeled with a uniform concrete wall subjected to the harmonic ground acceleration. The width of wall has been chosen in the way that its natural frequencies became equal to that of a benchmark numerical example Pine Flat gravity dam. In this regard, utilizing wall instead of a gravity dam is tenable. The height of the wall is 100 meter and has the width of 30 meter, unit weight is 24.8 KN/m^3, Poisson's ratio is 0.2, and modulus of elasticity is 22.75 GPa. The wave velocity in fluid is 1440 m/s^2.

A. Piston Type Motion of Rigid Wall

The rigid wall subjected to harmonic ground acceleration with amplitude of 0.109g, $a=|\ddot{U}_g|=0.109g$ in upstream direction, and frequency excitation (ω) of 6.2831rad/s is considered. The near-field sub-domain has the length of L=60m and is discretized by 450 4-noded elements as shown in the figure 2 also the boundary condition on interface 2 is descritized by 30 2-noded SBFEM elements . The piston type movement is a kind of motion in which all points of the wall are allowed to move horizontally with the same magnitude. The hydrodynamic pressure along wall-reservoir interface resulted from FEM-SBFEM and analytical solution [3] is plotted in figure 3.

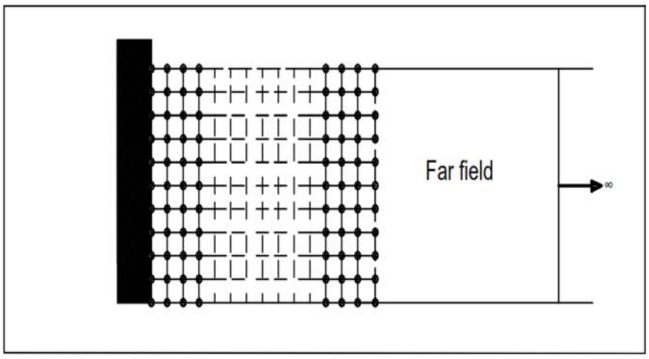

Fig 2.Meshes of near-flied sub-domain

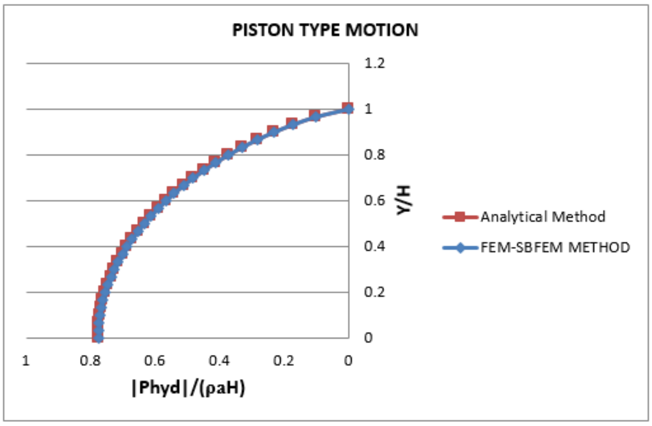

Fig 3.Hydrodynamic pressure on wall-reservoir interface, piston type motion

B. Flap Type Motion of Rigid Wall

In this type of motion all the points of the wall move horizontally but with the different distances corresponding to their heights as illustrated in figure 4. This movement of interface is considered as forced boundary condition, and the displacement of wall's crest is assumed to be$(〖U〗_c)\bar{}=0.02$m. The movement of wall's crest is expressed as$U_c=(U_c)\bar{}e^{i\omega t}$, and $a=\omega^2(U_c)\bar{}$. The analytical solution of hydrodynamic pressure computation at wall-fluid interface is discussed in Ref.[4]. Other assumptions are similar to former example. The hydrodynamic pressure acting on wall-reservoir interface resulted from FEM-SBFEM and analytical solution is plotted in figure 5. As illustrated in the figure, the slight difference in some elevations between the hydrodynamic pressure resulted from the two methods is because in the analytical solution the water is assumed to be incompressible but in the adopted method the water is linearly compressible.

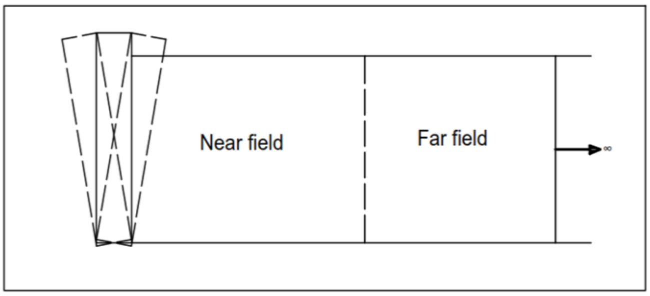

Fig 4.Flap type motion of wall, flap type motion

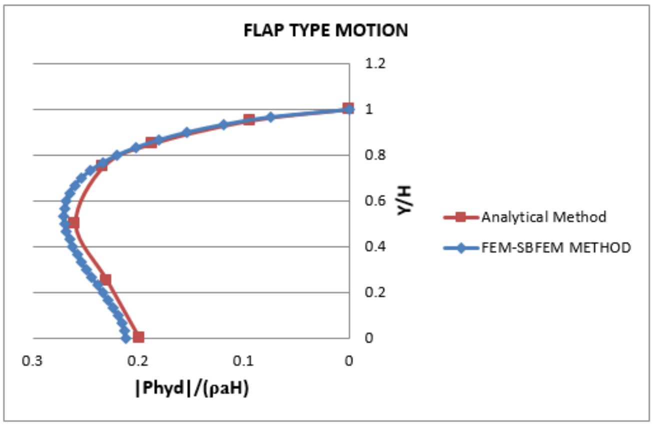

Fig 5.Hydrodynamic pressure on wall-reservoir interface, first mode type motion

C. Wall's First Mode Type Motion of Deformable Wall

Herein the wall is considered to be deformable and have the first mode type movement; indeed, this wall's deformation in obtained from the solution of Eigen-Value problem of structure while the reservoir is empty, and the horizontal displacements of all structure's points are scaled to displacement of wall's crest, that is,(U_c)̅=0.02m; In addition, a is defined as former example. In this regard, this type of motion is also forced boundary condition. The hydrodynamic pressure on fluid-structure interface is plotted in figure 6.

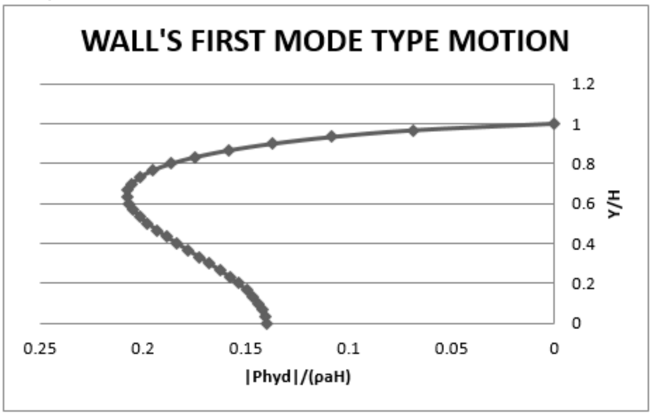

Fig 6.Hydrodynamic pressure on wall-reservoir interface

V. CONCLUSION

In this study, the effect of various structure's motions as forced boundary condition on hydrodynamic pressure has been investigated. As shown in the figures 5 and 6, the maximum hydrodynamic pressure has decreased by 30% when the wall is flexible and assumed to have first-mode type motion. Moreover, in the deformable wall's motion, the point in which maximum pressure occurs is in higher elevation than flap-type motion. The consistency between coupled FEM-SBFEM method and analytical solutions reveals that the accuracy of the mentioned method is remarkable also the computational cost owing to the necessity to only discretize the radiation boundary to model far-field sub-domain is considerably low.

This paper paves the way for future studies to compute the hydrodynamic pressure due to the earthquake excitation considering full fluid-structure interaction, in which the manifold modes of structure's deformation can be utilized readily.

REFERENCES

[1] Li S.M, Liang H, Li AM (2008) A semi-analytical solution for characteristics of a dam-reservoir system with absorptive reservoir bottom. Journal of Hydrodynamics, Vol.20, pp.727-734.

[2] Westergaard HM (1933) Water pressures on dam during earthquake. Transactions (ASCE), 98:418–433.

[3] Chopra A K(1967) Hydrodynamic pressures on dams during earthquakes. Journal of Engineering Mechanics, Div 93(6), 205–223.

[4] Dean R, Dalrymple R. Water Wave Mechanics For Engineers and Scientists. World Scientific Publishing Co.Pte.Ltd, 1984, p. 170-186.

[5] Fan SC, Li SM (2008). Boundary finite-element method coupling finite-element method for steady-state analyses of dam-reservoir systems. Journal of Engineering Mechanics – ASCE, Vol.134, pp.133-142.