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Waiting Time Analysis of a Multi-Server System in an Out-Patient Department of an Hospital

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Abstract — **The Objective of the Study is to analyze the Queuing measures which help the Out-Patient Department in an hospital to function effectively Waiting on a Queue is not usually interesting, but reduction in this waiting time usually requires planning and extra investments. Queuing is a major challenge for Healthcare services all over the world, but particularly so in developing countries. Queuing theory is the branch of Operations Research in Applied Mathematics and deals with phenomenon of waiting lines. The Health Systems should have an ability to deliver safe, efficient and smooth services to the Patients. In this paper, we discuss about an Out-Patient Department in an Hospital and calculate the performance measures of the Queuing System based on the values of the parameters, for an Out-Patient Department in an hospital and we analyze how for the model under Study influence the behavior of the System.**

Index terms- **M/M/C Queuing Model, Waiting Time, Poisson Arrival, Performance Measures, Queuing Analysis, Steady-State distribution.**

I. INTRODUCTION

In the early 20th century, Queuing Theory originated from the Danish Engineer Erlang's study of Telephone exchange efficiency of Communication System. After the Second World War, especially with the rapid development of Computer and Communication technology, Queuing Theory got attention and developed fast. Also, it becomes an important branch of Operations Research and its corresponding disciplines, theory were developed [4]. Queuing system is a System consisting of flow of Customers requiring Service, where there is some restriction in the Service rate, that can be provided. Three main elements are commonly identified in any Service Centre namely; Population of Customers, the Service Facility and the Waiting lines [4, 5]. Queuing is a Challenge for all Healthcare Systems. In the developed World, considerable Research has been done on how to improve Queuing systems in various hospital settings [2]. Waiting on a Queue is not usually interesting, but reduction in this Waiting Time usually requires Planning and extra Investments. Queuing Theory involves the Mathematical Study of Waiting Lines. Out-Patient Department (OPD) services are one of the important aspects of Hospital administration. OPD is the mirror of the Hospital which reflects the performance of the Hospital being the first point of contact between the Patient and the Hospital Staff [1]. Queuing Theory consists of three parts: Input Process, Queuing Rates and Service Agencies [9].

In this paper, Queuing theory is used to develop an Out-Patient Department in an Hospital Scheduling System that give acceptable results for the Patients and the Staff to improve the Service facility and decrease the Waiting Time and thus perfecting the Patient Care in the OPD. The paper is organized as follows: In section 1, introduction. In section 2, the basic concepts are discussed. In section 3, we discuss the performance Measures of the model. In section 4, the results are tabulated and Comparison is done. In section 5, we give the conclusion of our model.

II. BASIC CONCEPTS OF THE MODEL

A. Queuing Theory

Queuing theory is Mathematical Modeling of Waiting lines, whether of people, signals, or things. It uses models to represent the various types of Queuing Systems. Formula for each model indicates how the related Queuing System should perform under the variety of conditions.

B. Terminology and Notations

The following Terminology and notations are used in the Model Formulation and calculations.

P_n − Probability of exactly n Customers in the System

N − Number of Customers in the System

L_q −Expected (average) number of Customers in the Queue

L_s − Expected number of Customers in the System

W_s − Waiting time of Customers in the System

W_q −Waiting time of Customers in the Queue

λ_n − The Mean Arrival Rate (expected number of arrivals per unit time) of new Customers in System.

μ_n − The Mean Service Rate for over all systems (Expected Number of Customers completing Service per unit time) when n Customers are in Systems.

$\rho = \frac{\lambda}{c\mu}$ – Utilization factor for the service facility of Multi server model (when $n \geq C$, $\mu = C\mu$, i.e all C servers are busy)

C. M/M/C:(∞ / FIFO) MODEL

This Model treats the condition in which there are several Stations in Parallel and each Customer in the Waiting Queue can be served by more than one Station channel. Consider the M/M/C Queue with Arrival rate λ, Service rate μ and c Servers. The traffic intensity is defined usually by the ratio $\rho = \frac{\lambda}{c}\mu$. The Steady State distribution of Queuing System [5] is studied in the following:

$$P_n = P(N = n),\ n = 0, 1, 2, \dots$$

is the Probability Distribution of the Queue Length N, as the System is in Steady State, when the number of System Servers is C, then we have $\lambda_n = \lambda$, n = 0,1,2,…

If there are n Customers in the Queuing System at any point in time, then the following two cases may arise.

a. If n < C (Number of Customers in the System is less than the Number of Servers) ,then there will be Queue. However, (C - n) number of servers will not be busy. The combined service rate will be then $\mu_n = n\mu$; n < C.

b. If n ≥ C (Number of Customers in the System is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the Queue will be (n – C). The combined service rate will be $\mu_n = C\mu$; n ≥ C. From the model the probability of having n Customers in the System is given by

$$\rho = \lambda / C\mu \tag{1}$$

Solving for the Steady-State distribution yields

$$P_n = \begin{cases} (\rho^n/n!)P_0 & n \le c \\ \rho^n/(C!\,C^{n-c})P_0 & n > c \end{cases}$$

Which yields,

$$P_0 = \left(\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu-\lambda}\right)^{-1} \tag{2}$$

Expected number of the Customers waiting on the Queue:

$$L_q = \left[\frac{1}{(c-1)!}\left(\frac{\lambda}{\mu}\right)^c \frac{\mu\lambda}{(c\mu-\lambda)^2}\right]P_0 \tag{3}$$

Expected number of Customers in the System:

$$L_s = L_q + \frac{\lambda}{\mu} \tag{4}$$

Expected Waiting time of the Customers in the Queue:

$$W_q = \frac{L_q}{\lambda} \tag{5}$$

Expected time a Customer spends in the System:

$$W_s = \frac{L_s}{\lambda} \tag{6}$$

III. PERFORMANCE MEASURES OF THE MODEL

This Section deals with the analysis of data collected from all the three sections, in an Out-Patient Department of a Private Hospital.

A. Registration
B. History and
C. Consulting room

A. Registration Section

Let us consider the values λ, μ and C as follows
Mean Arrival rate (λ) = 100 patients per hour
Service rate (μ) = 55 patients per hour
Total number of server (C) = 3

1. From equation (1), the utilization factor for the Entire System is;

$$\rho = \frac{100}{3(55)}$$

$$= 0.60 \approx 60\%$$

2. From the equation (2), the Probability that the System would be idle is;

$$P_0 = \left(1 + \frac{100}{55} + \frac{1}{2!}\left(\frac{100}{55}\right)^2 + \frac{1}{3!}\left(\frac{100}{55}\right)^3 \frac{3\times 55}{3\times 55 - 100}\right)^{-1}$$

$$= 0.1425 \approx 14.25\%$$

3. From the equation (3), the Expected number of Patients waiting in the Queue is;

$$L_q = \left(\frac{1}{(3-1)!}(1.8181)^3 \frac{55\times 100}{(3\times 55 - 100)^2}\right)0.1425$$

$$= 0.5573 \approx 1 \text{ patient}$$

4. From the equation (4), the Expected number of Patients Waiting in the System is;

$$L_s = 0.5573 + 1.8181$$

$$= 2.3754 \approx 2 \text{ patients}$$

5. From the equation (5), the Expected time, the Patients spends in the System is;

$$W_q = \frac{0.5573}{100}$$

$$= 0.0055 \approx 0.33 \text{ minutes}$$

6. From the equation (6), the Expected Waiting time of a Patient in the Queue is;

$$W_s = \frac{2.3754}{100}$$

$$= 0.0237 \approx 1.422 \text{ minutes}$$

The results show that the Server would be busy 60% of an hour and idle 14.25% of an hour. Also, the average number of Patients Waiting in the Queue is 1 and the average number of Patients Waiting in the System is 2. More so, the average time a Patient spends in the Queue is 0.3 minutes and average time a Patient spends in the system is 1.4 minutes.

B. History Section

Mean Arrival rate (λ) = 100 patients per hour
Service rate (μ) = 40 patients per hour
Total number of server (C) = 3

1. From equation (1), the utilization factor for the Entire System is;

$$\rho = \frac{100}{3(40)}$$

$$= 0.8333 \approx 83\%$$

2. From the equation (2), the Probability that the System would be idle is;

$$P_0 = \left(1 + \frac{100}{40} + \frac{1}{2!}\left(\frac{100}{40}\right)^2 + \frac{1}{3!}\left(\frac{100}{40}\right)^3 \frac{3\times 40}{3\times 40 - 100}\right)^{-1}$$

$$= 0.0449 \approx 4.49\%$$

3. From the equation (3), the Expected number of Patients waiting in the Queue is;

$$L_q = \left(\frac{1}{(3-1)!}(2.5)^3 \frac{40\times 100}{(3\times 40 - 100)^2}\right) 0.0449$$

$= 3.5078 \approx 4$ patients

4. From the equation (4), the Expected number of Patients Waiting in the System is;

$$L_s = 3.5078 + 2.5$$

$= 6.0078 \approx 6$ patients

5. From the equation (5), the Expected time, the Patients spends in the System is;

$$W_q = \frac{3.5078}{100}$$

$= 0.0350 \approx 2.1$ mintues

6. From the equation (6), the Expected Waiting time of a Patient in the Queue is;

$$W_s = \frac{6.0078}{100}$$

$= 0.0600 \approx 3.6$ mintues

The results show that the Server would be busy 83% of an hour and idle 4.49% of an hour time. Also, the average number of Patients Waiting in the Queue is 4 and the average number of Patients Waiting in the System is 6. More so, the average time a Patient spends in the Queue is 2.1 minutes and average time a Patient spends in the System is 3.6 minutes.

C. Consulting Room

Mean Arrival rate (λ) = 100 patients per hour
Service rate (μ) = 35 patients per hour
Total number of server (C) = 4

1. From equation (1), the utilization factor of the Entire System is;

$$\rho = \frac{100}{4(35)}$$

$= 0.7143 \approx 71\%$

2. From the equation (2), the Probability that the System would be idle is;

$$P_0 = \left(1 + \frac{100}{35} + \frac{1}{2!}\left(\frac{100}{35}\right)^2 + \frac{1}{3!}\left(\frac{100}{35}\right)^3 + \frac{1}{4!}\left(\frac{100}{35}\right)^4 \frac{4\times 35}{4\times 35 - 100}\right)^{-1}$$

$$= 0.0464 \approx 4.64\%$$

3. From the equation (3), the Expected number of Patients Waiting in the Queue is;

$$L_q = \left(\frac{1}{(4-1)!}(2.8571)^4 \frac{35\times 100}{(4\times 35 - 100)^2}\right) 0.0464$$

$= 1.1272 \approx 1$ patient

4. From the equation (4), the Expected number of Patients Waiting in the System is;

$$L_s = 1.1272 + 2.8571$$

$= 3.9843 \approx 4$ patients

5. From the equation (5), the Expected time, the Patients spends in the System is;

$$W_q = \frac{1.1272}{100}$$

$= 0.0112 \approx 0.672$ mintues

6. From the equation (6), the Expected Waiting time of a Patient in the Queue is;

$$W_s = \frac{3.9843}{100}$$

$$= 0.0398 \approx 2.388 \text{ minutes}$$

The results show that the Server would be busy 71% of an hour and idle 4.64% of an hour. Also, the average number of Patients Waiting in the Queue is 1 and the average number of Patients Waiting in the System is 4. More so, the average time a Patient spends in the queue is 0.7 minutes and average time a Patient spends in the System is 2.4 minutes.

V. Results and Discussion

A. Comparison of the Data

Table 1: The Operating Characteristices at all the various sections

Operating Characteristices	**Registration**	**History**	**Consulting**
The mean arrival rate (λ)	100	100	100
The mean service rate (μ)	55	40	35
Utilization factor of the system (ρ)	60%	83%	71%
The probability that the system would be idle (P_0)	14%	4%	5%
The expected number of patients in the queue (L_q)	1 p	4 p	1 p
The expected number of patients in the system (L_s)	2 p	6 p	4 p
The average time a patient spends in the queue (W_q)	0.3 m	2.1 m	0.7 m
The average time a patient spends in the system (W_s)	1.4 m	3.6 m	2.4 m

Note – P is Patient and m is minutes

From above table 1, the busiest of all the sections is the History Section. Its utilization factor is 83% followed by the Consulting Room Section with utilization factor of 71% and the Registration Section recorded the least of 60%. The table also shows that the History Section has more Patients Waiting in the Queue than that of the Consulting Room and the Registration Section. On the average, a Patient spends about 3.6 minutes, in the entire system of the History Section and 2.4 minutes, in the Consulting Room and 1.4 minutes at the Registration Section.

B. Comparison Graph

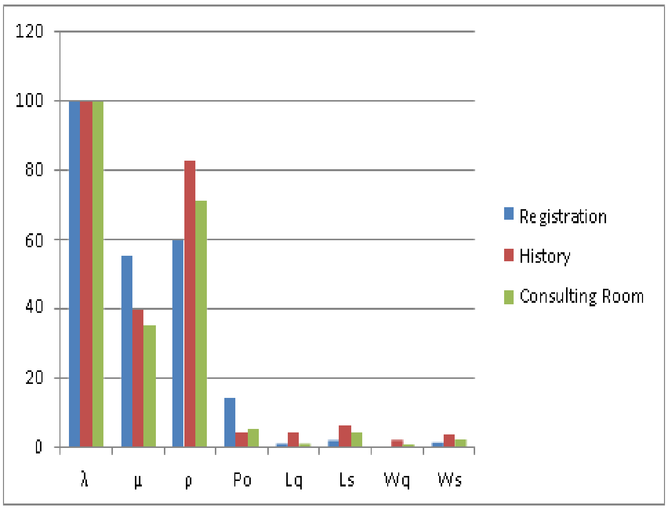

Fig.1 Performance Measures in the System

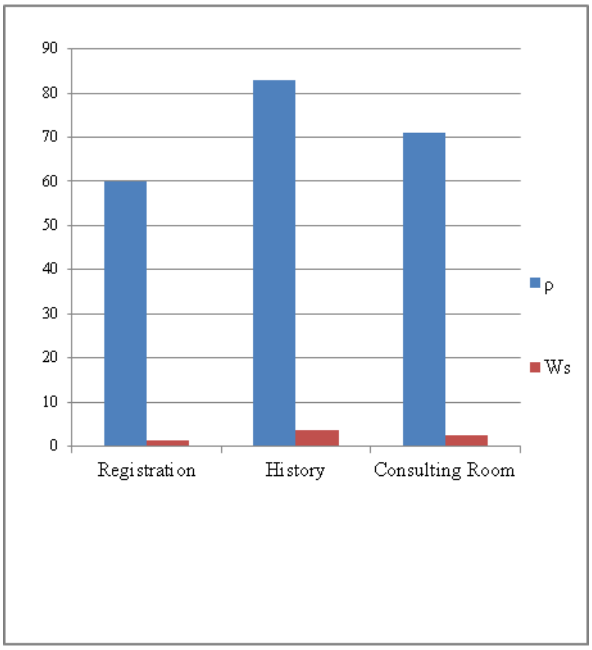

Fig.2 Utilization factor and analyze Waiting time in the system

V. Conclusions

The number of minutes a person spends before his or her history is taken from the History Section is far more than that of the other two sections. The Study of all various sections of a Private Hospital Out-Patient Department deals with Patient's Arrival rate, Service rate and the utilization factor of the whole system, where three parameters were then use to measure the waiting time of patients in the available Queues and in the Entire System. In total, it was found out that Patients had to wait for long in the Queues of the History and Consulting sections of the Private Hospital. Which shows that Registration section plays an effective role in Out-Patient Department of the Hospital.

REFERENCES

[1] Akua Amponsaa Tawiah, David Asamoah, John Mensah, Optimizing Patient Flow and Resource Utilization in Out Patient Clinic: A Comparative Study of Nkawie Government Hospital and Aniwaa Health Center, 2014- Vol.16(3), Kwame Nkrumah University of Science and Technology

[2] Alex Appah, Sam Afrane, Queuing theory and the management of Waiting-time in Hospital: The case of Anglo Gold Ashanti Hospital in Ghana, 2014-Vol.4,No.ISSN: 2222-699, Australia.

[3] Bhat U.N, An Introduction to Queuing Theory, USA, LLC-2008.

[4] Gross D, Shortle J.F, Thompson J.M, and Harris C.M.(Auth) Fundamentals of Queuing Theory 4th edition, John Wiley and sons-2008.

[5] Haribaskaran G, Probability, Queuing Theory and Reliability Engineering, Laxmi Publication, New Delhi-2005.

[6] Medhi J, Stochastic Processes, New Age International Publishers, Third edition, New Delhi-2009.

[7] Okoro Joshua Otobritise, Ogunlade Temitope Olu, Waiting Time Analysis of A Single Server Queue in an Out-Patient Clinic, 2015-Vol.11,No.ISSN: 2319-765X, PP 54-58, Nigeria.

[8] Robert B.Cooper, Introduction to Queuing Theory(NH=1981), New York-1981.

[9] Shuguno Yang, Xiaoyan Yang, The Application of the Queuing Theory in the Traffic Flow of Intersection, 2014-Vol.8, No.6, China.