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VIBRATION ANALYSIS OF STIFF PLATE WITH CUTOUT

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***Abstract*—the present paper deals with the analysis of vibration of a Square plate with circular hole. As the Plate structures are accounted as major engineering substructures, especially wherever the weight is a main issue. In aerospace and marine structures, the structural weight beside the necessary strength is of chief importance. Moreover, good strength-to-weight and stiffness-to-weight ratios as well as design versatility have made composite laminates very demotic engineering structures. The presence of singularity in form of hole affects the fundamental frequency of plate and hence the vibrational analysis of plate with hole becomes significant in pivotal installation. The present paper analyzes the effect of circular hole located at the centre of the plate on the natural frequency of square plate by using finite element method. Two boundary conditions are used, namely SSSS Simply supported all edges, CCFF two adjacent edges are clamed and two adjacent edges are free. The parametric study of the reliance of the hole diameter on the frequency and mode shape of a square plate are discussed and found that the natural frequency of plate decreases with increase in hole diameter, which is more appreciable in higher modes.**

***Index Terms*— Natural Frequency, Cutout, SSSS, CCFF.**

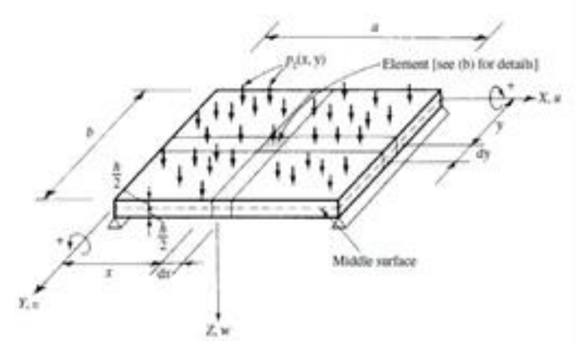
I. Introduction

Plates are straight, plane, two-dimensional structural components of which one dimension, referred to as thickness h, is much smaller than the other dimensions. Geometrically they are bound either by straight or curved lines. Like their counterparts, the beams, they not only serve as structural components but can also form complete structures such as slab bridges, for example Statically plates have free, simply supported and fixed boundary conditions, including elastic supports and elastic restraints, or, in some cases, even point supports The static and dynamic loads carried by plates are predominantly perpendicular to the plate surface. These external loads are carried by internal bending and torsional moments and by transverse shear forces. Since the load-carrying action of plates resembles to a certain extent that of beams, plates can be approximated by grid works of beams. Such an approximation, however, arbitrarily breaks the continuity of the structure and usually leads to incorrect results unless the actual two-dimensional behaviour of plates is correctly accounted for The two-dimensional structural action of plates results in lighter structures and, therefore, offers economical advantages. Consequently, plates and plate-type structures have gained special importance and notably increased applications in recent years. Since there are different types of plate such as stiff plates, Membranes, Moderately thick plates, Thick plates. As per which different plate theory has been given such as Kirchhoff's plate theory, Midlin plate theory, Membrane plate theory, and Classical plate theory , in these theory the plate behaviour is been discussed under different boundary condition along with different in plane loading condition and moments.

[1]G.J. Turvey , N. Mulcahy the Experiments have been carried out to determine the free vibration frequencies and mode shapes of 3.2 mm thick, pultruded GRP, square plates with six combinations of clamped (C), simply supported (S) and free (F) edge supports. Comparison of experimental and theoretical/numerical frequencies conferms that thin homogeneous orthotropic/anisotropic plate theory provides a reasonable model for predicting the free vibration response of pultruded GRP plates. [2]Lee H.P and S.P Lee analysed a simply supported square plate with a square cutout subjected to in-plane forces by sub-dividing the plate into sub-domains.by RR method.[3]Moon K. Kwak, Sangbo Han works on the vibration analysis of a rectangular plate with a rectangular hole or a circular hole using ICC method. [4]Lam divided the rectangular plate with a hole into several subareas and applied the modified RRM. Lam and Hung [5] applied the same method to a stiffened plate. The admissible functions used in Refs. [6,7] are the orthogonal polynomial functions proposed by Bhat [8,9] Rajamani and Prabhakaran [10]. assumed that the effect of a hole is equivalent to an externally applied loading and carried out a numerical analysis based on this assumption for a composite plate.[11]Ovesy H.P analyses Buckling and free vibration finite strip analysis of composite plates with cutout based on two different modeling approaches. [12]K.Y Lam, K.C hung works on efficient and accurate numerical method in the study of the vibration of rectangular plates with cutouts and non-homogeneity is presented.

II. Mathematical Modelling

A mathematically exact stress analysis of a thin plate—subjected to loads acting normal to its surface—requires solution of the differential equations of three-dimensional elasticity [1.1.1].Let considered an rectangular plate with side a in X direction and b in Y direction and transverse deflections w(x, y) for which the governing differential equation is of fourth order, requiring only two boundary conditions to be satisfied at each edge.



(a)
Laterally loaded rectangular plate

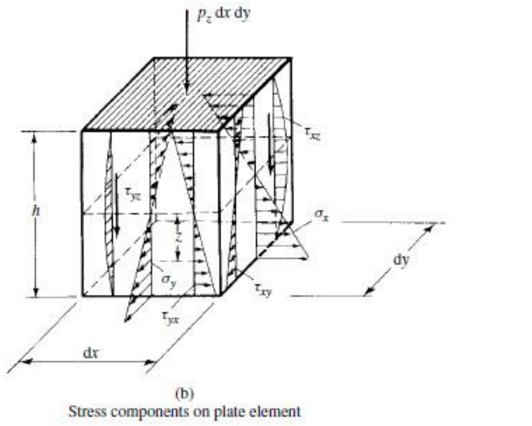


Figure1 Shows laterally loaded rectangular plate

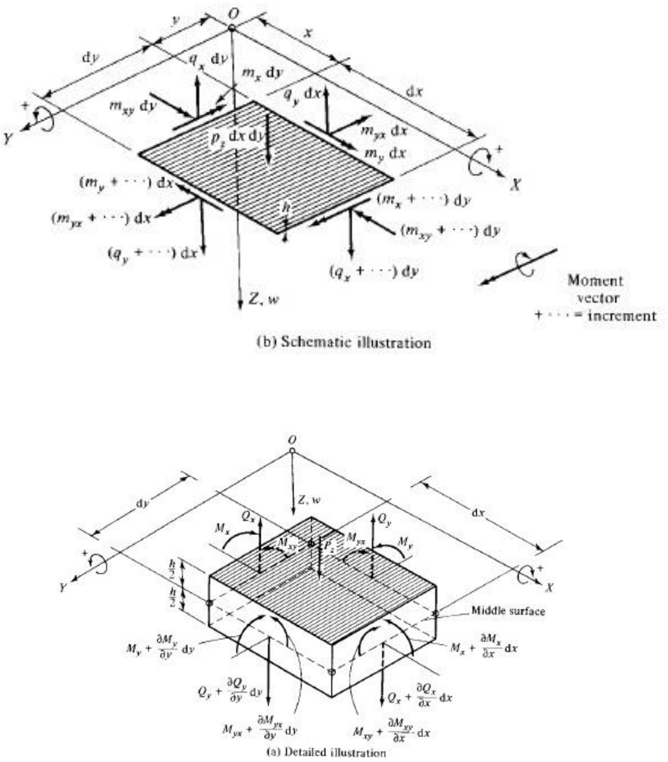


Figure 2 Shows External and internal forces on the element of the middle surface

Plate Equation in Cartesian coordinate system

Let us express, for instance, that the sum of the moments of all forces around the Y axis is zero. The governing differential equation for thin plates, we employ, for pedagogical reasons, a similar method as used in elementary beam theory.

Equilibrium of Plate Element

$$\sum M_x = 0, \ \sum M_y = 0 \quad \sum p_z = 0 \tag{1}$$

$$(m_x + \frac{\partial m_x}{\partial x}dx)\,dy - m_x dy + (m_{yx} + \frac{\partial m_{yx}}{\partial x}dy)\,dx$$
$$-m_{yx}dx - (q_x + \frac{\partial q_x}{\partial x}dx)\,dy\frac{dx}{2} - q_x dy\frac{dx}{2} = 0 \tag{2}$$

After simplification, we neglect the term containing since it is a small quantity of higher-order. Thus Eq. (2) becomes ½(∂qx/∂x)(dx)2dy since it is a small quantity of higher-order. Thus Eq. (2) becomes

$$\frac{\partial m_x}{\partial x}dxdy + \frac{\partial m_{yx}}{\partial y}dydx - q_x dxdy = 0 \tag{3}$$

$$\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -p_z(x,y). \tag{4}$$

The bending and twisting moments in Eq. (4) depend on the strains, and the strains are functions of the displacement components (u, v,w)

Relation between Stress, Strain and Displacements

$$\sigma_x = E\varepsilon_x + \nu\sigma_y,$$
$$\sigma_y = E\varepsilon_y + \nu\sigma_x,$$
$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right),$$
$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) \tag{5}$$

The torsional moments m_{xy} and m_{yx} produce in-plane shear stresses τ_{xy} and τ_{yx} (Figure. 1), which are again related to the shear strain γ by the pertinent Hookean relationship, giving

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} = \tau_{yx} \tag{6}$$

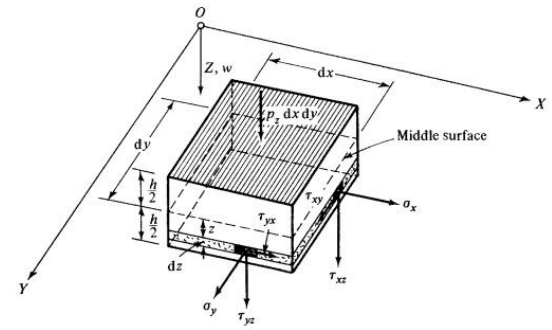


Figure 3 Shows Stresses on a plate element.

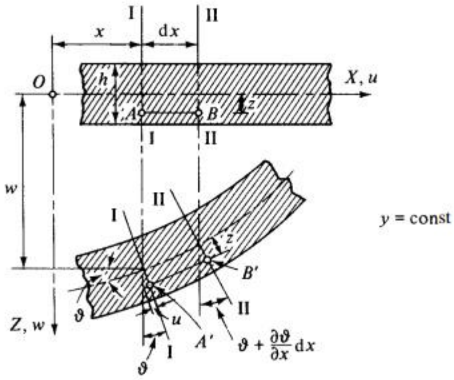


Figure 4 Shows Section before and after deflection

The curvature changes of the deflected middle surface are defined by

$$\kappa_x = \frac{\partial^2 w}{\partial^2 x}, \quad \kappa_y = -\frac{\partial^2 w}{\partial^2 y} \quad \text{and} \quad \chi = -\frac{\partial^2 w}{\partial x \partial y} \tag{10}$$

Internal Forces Expressed in Terms of w

$$m_x = \int_{-(h/2)}^{+(h/2)} \sigma_x z dz \;, \; m_y = \int_{-(h/2)}^{+(h/2)} \sigma_y z dz \tag{11}$$

$$m_{xy} = \int_{-(h/2)}^{+(h/2)} \tau_{xy} z dz \quad m_{yx} = \int_{-(h/2)}^{+(h/2)} \tau_{yx} z dz \quad , \tag{12}$$

$$m_x = -\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^2 w}{\partial^2 x} + \nu \frac{\partial^2 w}{\partial^2 y}\right) \quad m_y = -\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^2 w}{\partial^2 y} + \nu \frac{\partial^2 w}{\partial^2 x}\right), \tag{13}$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{14}$$

Represents the bending or flexural rigidity of the plate. In a similar manner, the expression of twisting moment in terms of the lateral deflections is obtained

$$m_{xy} = m_{yx} = \int_{-(h/2)}^{+(h/2)} \tau z dz = -2G \int_{-(h/2)}^{+(h/2)} \frac{\partial^2 w}{\partial x \partial y} z^2 dz$$

$$= -(1-\nu) D \frac{\partial^2 w}{\partial x \partial y} = D(1-\nu)\chi. \tag{16}$$

governing differential equation of the plate subjected to distributed lateral loads,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{Pz(x,y)}{D} \tag{17}$$

III. Finite Element Results

In this paper an isotropic Square plate made of Aluminum having E= 7.3 × 10^{10}, ρ = 2880kg/m^3,ν =0.33 is analyzed for its vibration characteristics using commercially available finite element code Ansys V12

A 3 dimensional plate of thickness h and dimension axb is modeled in mechanical APDL, solid 186 element is used for meshing and sweep meshing is done and convergence is reached using h method. Model analysis is carried out for various increasing diameter of the hole.

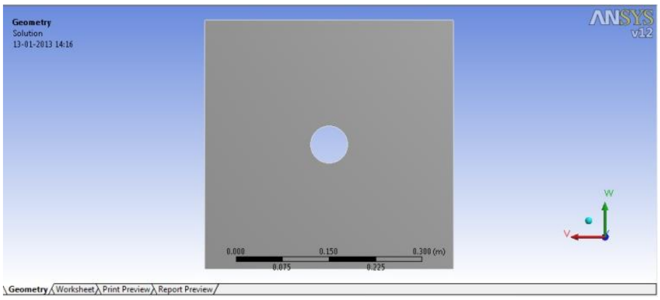


Figure 5 shows the plate model with centrally located hole, boundary condition SSSS

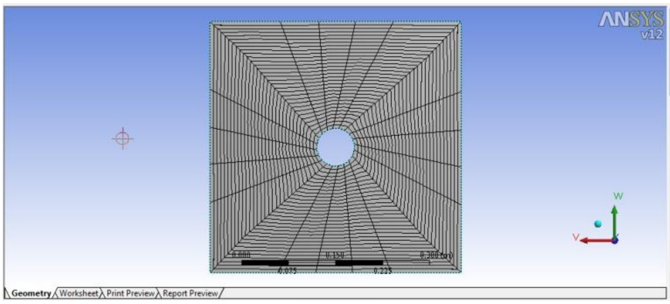


Figure 6 shows the mesh plate model with centrally located hole, boundary condition SSSS

Table 1. Shows 1st mode of natural frequency for isotropic square plate with aspect ratio a/b=1, with hole

Natural Frequency of plate with circular cutout in (Hz) a=b= 0.4,t=0.003,D=0.06				
	SSSS			CCFF
	Without Hole	**Present**	**Reference[3]**	**Present**
Mode1	90.423	93.334	93.368	33.5
2	226.22	229.85	229.88	117.63
3	226.22	229.97	229.92	124.35
4	361.51	363.53	363.59	240.03
5	452.62	450.72	450.768	298.95
6	452.63	460.66	460.768	319.28

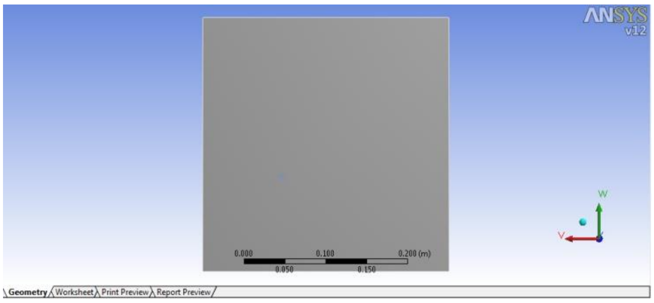


Figure 7 shows the plate model boundary condition SSSS

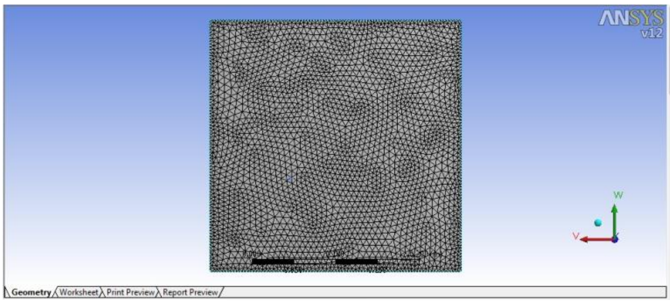


Figure 8 shows the refine mesh plate model, boundary condition SSSS

Table 2. Shows 1st mode of natural frequency for isotropic square plate with aspect ratio a/b=1, without hole

Natural Frequency of plate in SSSS a=b= 0.3,t = 0.0032			
	SSSS		CCFF
	Present	**Reference[1]**	**Present**
Mode1	174	174	61.201
2	435.49	435.6	211.36
3	435.51	435.6	235.23
4	695.66	695.6	420.85
5	871.56	872.3	554.64
6	871.56	872.3	579.76

Mode Shapes

Mode shape generate during free vibration of plate are as follows.

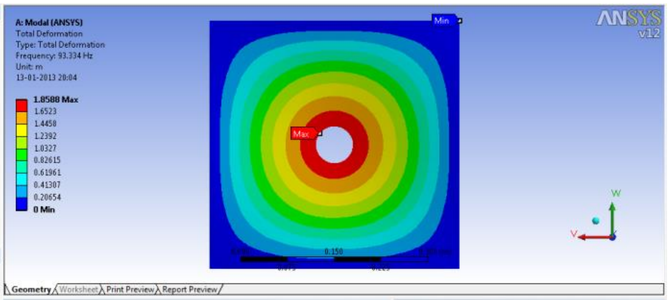


Figure 9 shows the 1st mode shape of centrally located hole , boundary condition SSSS

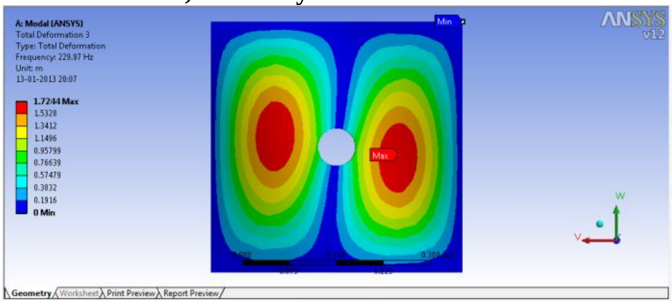


Figure 10 shows the 3st mode shape of centrally located hole , boundary condition SSSS

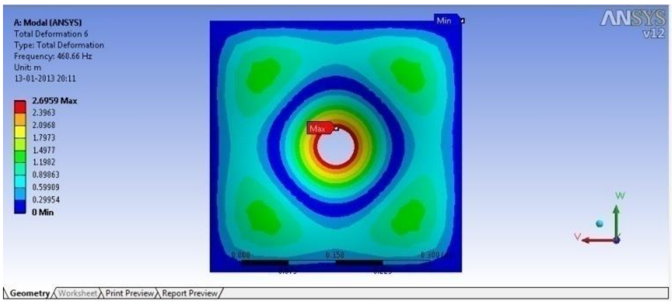


Figure 11 shows the 6st mode shape of centrally located hole , boundary condition SSSS

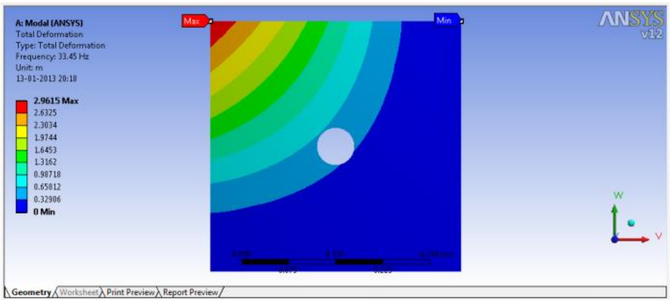


Figure 12 shows the 1st mode shape of centrally located hole , boundary condition CCFF

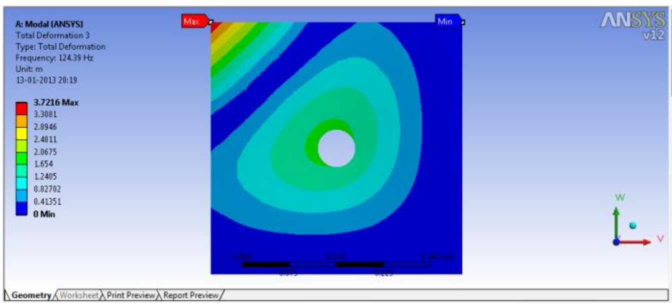


Figure 13 shows the 3st mode shape of centrally located hole , boundary condition CCFF

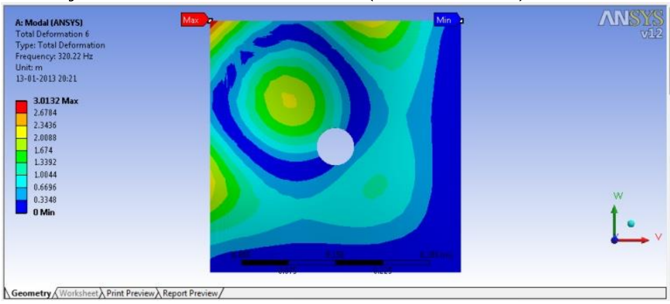


Figure 14 shows the 3st mode shape of centrally located hole , boundary condition CCFF

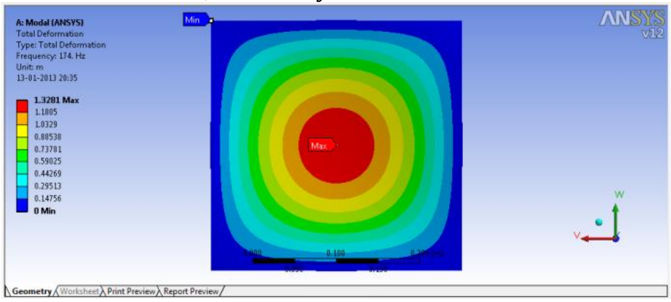


Figure 15 shows the 1st mode shape of plate, boundary condition SSSS

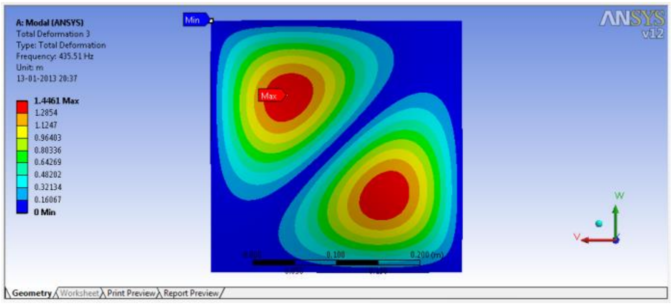


Figure 16 shows the 3rd mode shape of plate, boundary condition SSSS

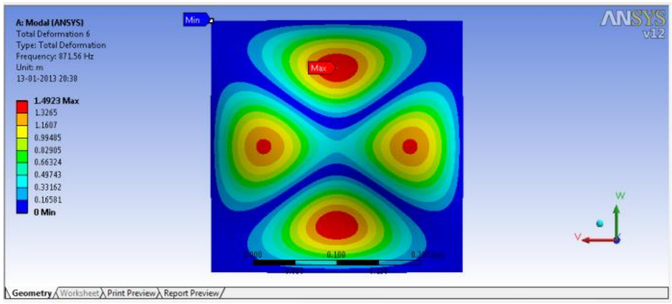


Figure 17 shows the 6th mode shape of plate, boundary condition SSSS

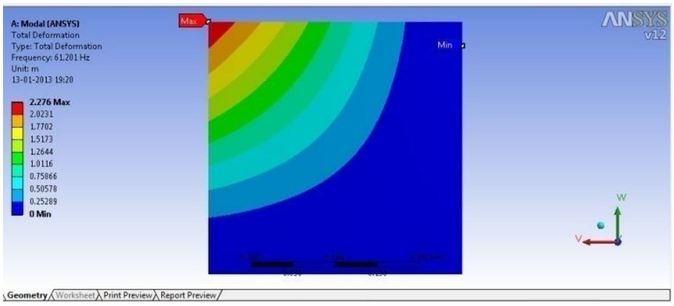


Figure 18 shows the 1st mode shape of centrally located hole , boundary condition CCFF

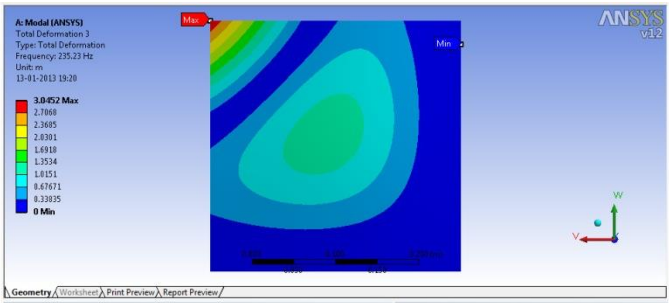


Figure 19 shows the 3rd mode shape of centrally located hole , boundary condition CCFF

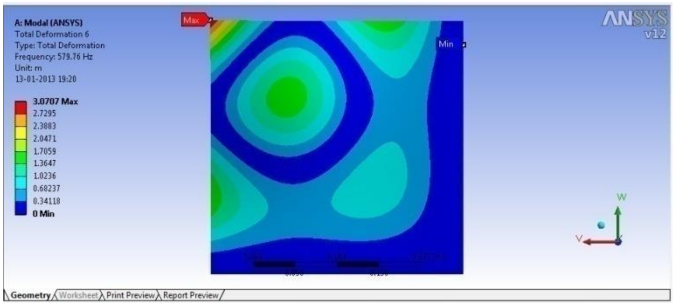
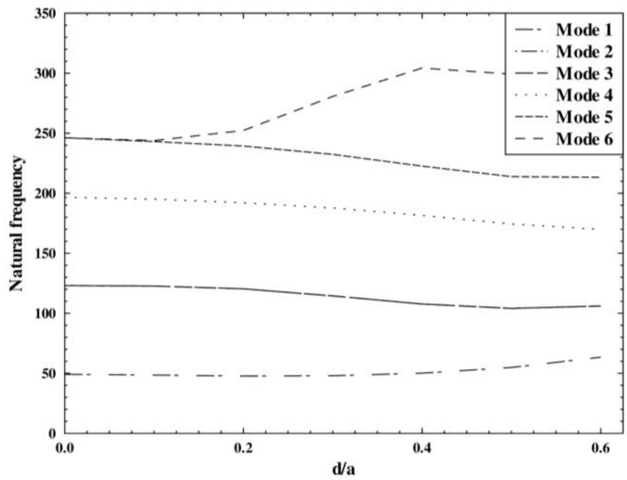
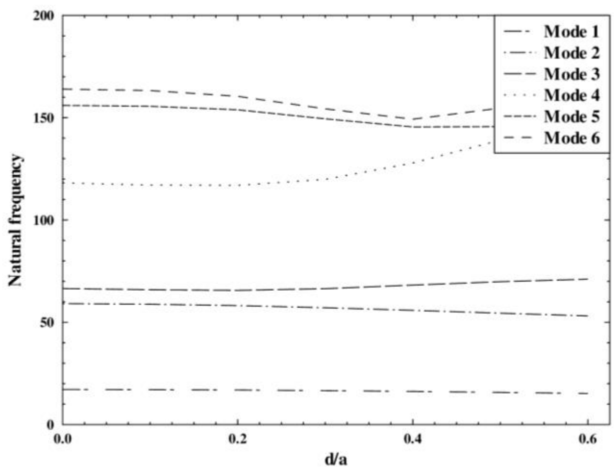


Figure 20 shows the 6st mode shape of centrally located hole , boundary condition CCFF



Graph 1 shows non dimensional frequency Vs d/a plate boundary condition SSSS



Graph 2 shows non dimensional frequency Vs d/a plate boundary condition CCFF

Figure 5-8 shows the square plate model with and without hole and its meshing upon which the analysis has been carried out to find natural frequency of vibration under SSSS and CCFF .

Figure 9-20 shows the mode shapes under SSSS and CCFF of above mentioned model and the result obtained is validated with reference[3] and reference[1] as tabulated in table 1 and 2.

The graph 1 shows non dimensional frequency Vs d/a plate boundary condition SSSS from this graph it is observed that natural frequency in all the modes of vibration remains all most constant up to d/a 0.1 and then changes rapidly. And it is also observed that mode 2 and 3 remain constant throughout in SSSS Boundary condition, mode 4 and 5 the frequency starts decreasing after d/a =0.4,while mode 6 first decreases and increases up to D/a=0.4 and again starts decreasing.

Graph 2 shows non dimensional frequency Vs D/a ratio plate boundary condition CCFF from this graph it is observed that natural frequency of plate in mode 1 remain almost constant or little bit variation is been seen. In 2 mode the frequency remain constant up to d/a = 0.2 and then afterward starts decreasing, similarly in mode 3 the frequency remain constant up to d/a = 0.25 and then afterwards starts increasing. Where in mode 4 the curve remain constant till d/a= 0.2-0.25 and then after d/a= 0.25 the frequency starts increasing up to d/a=0.5 and then afterwards starts decreasing. While at last mode 5and 6 frequency deceases with increasing d/a ratio up to d/a= 0.4 and then mode 5 increase slightly while mode 6 frequency increases rapidly .

From the above specific condition i.e SSSS and CCFF it is observe that deflection of plate in more than plate without hole, for SSSS, principally there is no difference in mode shape for plate with and without hole. Since from above results it is observed that presence of hole changes the stiffness and mass, and hence the natural frequency. The extensive results given show this variation for 6 mode shapes and 2 boundary conditions, namely CCFF and SSSS. The presence of hole affects more, the natural frequency of SSSS plate as compare to CCFF.

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