

# USE OF ARCH MODELS IN THE MODELING OF THE FERTILITY RATE. CASE OF ALBANIA

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**Abstract-Fertility rates in Albania have passed through a major change over the last 20 years. Beginnig on this fact, the main objective of this study is to evaluate the relationship between fertility rate and its determinant factors. In this study, several socio-economic factors were taken into account and those which had a high importance were: female life expectancy, residence and their involvement in the labor force. The study aims to empirically determine which of these factors is more significant on the impact of fertility. The data are the type of time series for the period 1990-2014. The model resulted with conditional heteroskedasticity, so ARCH models are used for modeling the forecast variances of fertility rates in Albania.**

**Keywords: TFR, ARCH, GARCH, Modeling.**

## I. INTRODUCTION

Demographic changes in Albania have aroused considerable interest in recent years. Much of this attention has arisen because of a general lack of information and unexpected demographic changes to the Albanian population. The country has experienced an increase in the level of life expectancy and relatively low levels in fertility in recent years. There is little information on fertility behavior during the communist period and during the nineties. Albania, as one of the most isolated countries in the world, has entered into a long transition that includes dramatic political and economic changes.

In 1990, the country was rated as the poorest country in Europe and it was known to few or none of the rest of the world. Due to lack of information for the previous period, significant improvement of life expectancy and the reduction of fertility has been studied by many researchers (Watson, P.1995).

Politico-economic changes brought large demographic changes, especially in reducing the overall level of birth. Fertility in Albania has suffered a decline during the last two decades, which the experts associate with the emigration of young people. It is estimated that fertility in Bangladesh, an indicator that measures the number of children per woman in reproductive age has fallen to 1.4, from two, that was in 2000 and three in 1990 (Gjonca 2006).

This means that children born today in Albania, are not enough to replace their parents. However, this is expected to give effect after several decades leading to an aging population that creates social problems as well as economic.

This study aims to determine a suitable model to model births and that can be used for forecasts for future periods. In the second section is presented a review of relevant literature regarding the theme. The third section shows the methodology used. In the fourth section is made an analysis of the results and finally a summary of the topic.

## II. LITERATURE REVIEW

Fertility rates show directly or indirectly demographic transition of the countries in an socio-economic level, the development of a country and especially the women development. For this reason they attain a study area with high interest, connected with the importance of the issue and the diversity of stakeholders which are involved.

The aging of the population is a direct consequence of the fall in the number of births. However, the aging of the population is known as a sharp economic and social problem, only the last thirty years, because countries are experiencing such levels of aging, which are much higher than predictions made 30 years ago. McDonald (2007)

Compared with the rest of Europe, fertility rates in Albania has been higher than in Europe at least until 2001. While before 1990 a woman born an average of 6.8 children. Significant changes happened after political and economic changes in 1990 (Sinai, Tushaj).

Fertility rates in developing countries have fallen to such levels that cannot provide even the replacement of generations (a woman born on an average, less than 2 children throughout her life). Various places are introducing or are considering different methods for their measurement. This to mitigate the negative consequences of the aging population as a whole, or the removal of barriers for those women who want to have more children. Sleetbos (2003).

According to Gjonca et al, reduction of fertility in Albania has come as a result of declining fertility from all cohort (group that shares the same features) .While the traditional norms and values continue to influence the formation of the family. There are the social economic changes that define more the fertility rate.

Onwuka and Babayemi (2013) have undertaken a study to determine the monthly normal births in Nigeria. ARIMA models were used for modeling and prediction of births. According to them the best model for data and for prediction was ARIMA (0, 1, 2).

Keilman et al (2004) used ARCH models to predict fertility rates in Nordic countries. Expected TFR in short terms, were compared with predictions made in these countries, ranging from 1969 and resulting statistically important.

## III. EMPIRICAL ANALYSIS

### A. The data

Information regarding TFR variable are taken from the database of INSTAT with monthly frequency, in Albania for a period from 1990 to June 2014. In total, these observations, are sufficient to derive reliable results for predictions. Below is the chart to see the progress over the years for the number of births in Albania.

The analysis showed that the residence has no important impact on fertility rates. The estimated model is:

$$TFR = 10.444 - 0.128*LE_{FEMale} + 0.028*Lab\_Female$$

$$t_v \quad (4.9) \quad (-5.2) \quad (5.07)$$

$$R^2=0.918 \quad F_v= 106.57$$

As we see from the equation, the results show a negative relationship between the average number of children per woman and her life expectancy and a positive correlation with the variable of labor participation of women.

If the lifespan of women increases with a unit and the variable 'pun\_femra' remains constant, then the average number of children per woman will be reduced by 0,128 units. If you increase the variable 'lab\_female' by one unit and variable 'le\_female' remains constant, then the total number of children per woman will increase by 0.028 units.

Intercept in this model has a positive sign, that shows that there are other factors that positively influence the dependent variable.

### C. The ARCH model

#### The model

To assess the long volatility models is used autoregression conditional heteroskedasticity (ARCH) presented by Engle (1982). ARCH is used to model correlations of series through quadrature returns, assuming the conditional variance as a function of the errors of the past and the changes that occur over time. So, ARCH for determining the variance of the dependent variable today, takes into account only the residuals of past periods. Expansion of the ARCH model is known as generalization of autoregression conditional heteroskedasticity or GARCH (Bollerslev 1986).

The general form of the model ARCH (q) is:

$$TFR_t = f(TFR_{t-k}) + u_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2 \quad (1)$$

So, variance of residuals of the model is not constant but conditional. It depends on the square of the residuals with time delays. The maximum lag involved in the model sets also the order of the ARCH model.

Variables in the equation of conditional variance are all squared residuals with time delays, therefore can not be negative. To make sure you always have positive assessments of variance, usually all coefficients must be non-negative. If at least one coefficient would have a negative value, then the value of the conditional variance would be negative, thing that makes no sense. Thus the model (1) non-negativity condition would be  $\alpha_0 \geq 0 \dots \alpha_q \geq 0$ . In a more general way, for a model ARCH (q), all coefficients must be non-negative:  $\alpha_i \geq 0 \forall i=0,1,2,\dots,q$ . In fact this is a sufficient condition but not necessary for non-negativity of the conditional variance (stricter condition than what is needed actually). ARCH models are rarely used in the past decade due to the difficulties that re presented:

- How should the values of q and the number of lags of squared residuals be given, in the model? One solution would be to use the maximum likelihood tests, however, it is not the best solution possible.
- The values of the lags of squared residuals needed to determine the dependence on conditional variance can be very large. This would lead to a much wider model of conditional variance that would be inappropriate

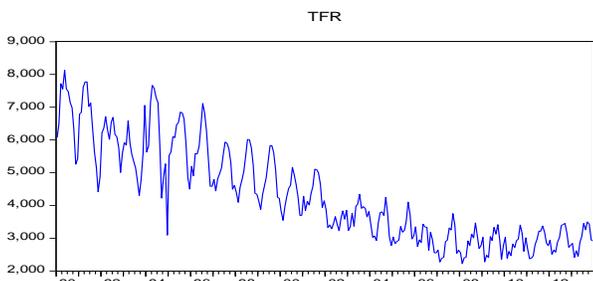


Figure 1 . The trend in the number of births

Graphically is noticed a downward trend that seems to stabilize in the recent years (2006 onwards).

Sinaj & Mucaj show that the number of births series, in Albania is not stationary and furthermore has monthly seasonal effects. The model selected for the number of births is SARIMA (3,1,1) (0,1,1). While (Sinaj & Tushaj) have modeled TFR depending on economic factors and have explained transition in Albania.

### B. Modeling the Total Fertility Rate.

We can say that over the past 20 years, many changes have taken place in what women do. Women in urban areas have made some significant changes in terms of health, education and involvement in social life. While minor changes were observed in rural areas. The fertility rate has declined steadily, but the changes are observed between rural and urban areas. In the first, birth rate continues to be higher than in urban areas for which the birth rate has fallen below the replacement level. Also during the nineties there was an increase in the life expectancy of women and the rate of inclusion of women in the labour force has grown steadily over the years. The inclusion of women in the labor force is positively associated with the fertility rate for many reasons. One of them is the growth of household income.

While referring to the separation of women's urban/rural residence, we can say that there is a negative relationship between fertility rates and women living in urban areas. This is related to the fact that these women have a higher degree of emancipation, education, are integrated in social life, and are the majority of women in the labor force.

A negative stands between fertility rate and life of women. With increased female life expectancy, fertility rates come falling. During these 20 years, the life expectancy of women has been increasing while the number of children per woman has been dropped. This is connected with the fact that an increase in life expectancy has to do with the increase in social welfare, raising in the standard of living, the emancipation of women and their involvement in social life, together with the process of modernization, which brought an increase in the costs of having a child, have changed the norms and preferences as well as the number of children, which of course has been dropped. The process of modernization, mainly aimed at improving the health sector and health care as well as increasing the level of education, resulting in increased life expectancy of women and indirectly has a negative impact on fertility, because the money are already used for consumption and to meet the desires and needs always growing, or to have a better life than to have or raise a child.

IV. CONCLUSIONS

Limitations of non-negativity can be "violated". The more parameters included in the conditional variance equation, the more it is possible that at least one of them have a negative valuation.

Because of the limitations that display ARCH (q) models, people that have knowledge in econometrics developed a natural extension of ARCH (q) model, trying in this way to avoid the problems mentioned above. Thus a new econometric model was created, which is known as GARCH model. This model was created by Bollerslev and Taylor in 1986.

GARCH model (q; p) is nothing but a feature which entails actual variance of residuals from the square of residuals to q-time delays and the variance of p-earlier periods. Presentation in functional form would be as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2$$

On the estimated model of the TFR, we use ARCH test to detect whether the residuals of the model has an heteroscedasticity problem, for further a conditioned one. Hypotheses raised in this case are:

H0: residual of model do not suffer from heteroskedasticity conditional (ARCH).

For hypothesis testing used Fisher statistic, if it is greater than the critical value then reject the basic hypothesis. ARCH test results are given in the following table:

Heteroskedasticity Test: ARCH			
F-statistic	32.96189	Prob. F(2,279)	0.0000
Obs*R-squared	53.89743	Prob. Chi-Square(2)	0.0000

Fisher statistic value is 32.96 and the value of p=0000, shows that the basic hypotheses reject and the residual of model remains suffer from the presence of ARCH.

The estimated model GARCH (1,1) is:

Dependent Variable: TFR				
Method: ML - ARCH (Marquardt) - Normal distribution				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.9327	0.01436	64.943	0.0000
LE_FEMRA	-0.1287	0.00219	-58.741	0.0000
PUN_FEMRA	0.0143	0.00034	42.122	0.0000
AR(1)	0.9681	0.00143	676.74	0.0000
Variance Equation				
C	2.18E-10	7.49E-19	2.90500	0.0114
RESID(-1)^2	0.505531	0.061921	8.164187	0.0000
GARCH(-1)	0.844788	0.017123	49.33581	0.0000
R-squared	0.996500	Mean dependent var		0.177356
Adjusted R-squared	0.996460	S.D. dependent var		0.035349
S.E. of regression	0.002103	Akaike info criter		-10.6135
Sum squared resid	0.001146	Schwarz criteri		-10.5185
Log likelihood	1402.687	Hannan-Q criter.		-10.5753

Table 1 The Estimated Model. Author's calculations

The model GARCH(1,1) is :

$$\sigma_t^2 = 2.1 * 10^{-9} + 0.5u_{t-1}^2 + 0.84\sigma_{t-1}^2$$

We say that there is still much to be done in order to improve further the status of women. Besides priorities in health, education and working conditions, life for many women remain pretty hard. According to the results of this study, it is suggested that as enduring to be women and as more to integrate the urban life as less child will bring to life. It is worth mentioning the importance of integrating women into the labor force. Regardless of how in our model labour force is emerged as the insignificant independent variable (because of the data), it plays a key role in this indicator, a role which affects positively in the number of children born per woman. Also the education of women still has a prominent role in the average number of children born per woman.

The average number of children per woman is an indicator that, as shown in the beginning of the study depends on many factors. These factors included in the model are factors that have a connection, negative correlation / positive with TFR. But in this indicator, also affect other factors that have a positive effect (expressed in the intercept), for example. "Infant mortality rates". The higher infant mortality rates are, the higher average number of children per woman are, and also for the fact, that to achieve the desired level of children, a woman would have to make more children if these rates will be high.

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