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STOCHASTIC ANALYSIS OF 2-OUT-OF-3 UNITS REDUNDANT SYSTEM WITH FINAL TESTING BEFORE USE

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***Abstract*— The objective of the present paper is to provide reliability analysis of 2-out-of-3 units of cylinders in an automobile engine by incorporating the idea of final testing before use for the repaired failed cylinder. The system of automobile engine consists of three identical cylinders. Initially all the three cylinders are operative and each of the unit has only two modes i.e. Normal and Complete failure. The system of automobile engine may operate satisfactorily if at least 2-out-of-3 cylinders are in operative mode but with the increased failure rate of the remaining cylinders. A single repair facility with discipline "FCFS" is considered. After each repair of failed cylinder it is sent for final testing where the repaired cylinder will be operative for a fixed period of time. If it works satisfactorily up to a fixed amount of time in testing then it goes for operation otherwise replace it by the new one. The probability that the repaired cylinder will found to be satisfactorily in testing is fixed. The failure time distribution of operative cylinder is exponential while the distribution of completing repair, final testing and replacement are general.**

***Index Terms*— Redundant System, Stochastic Analysis.**

I. INTRODUCTION

Various researchers engaged in the field of engineering reliability have developed several engineering models with the assumption that the system works satisfactorily if at least one of its unit is operative. But in many practical situations there exist some engineering systems which may operate if at least k out of n units are operative mode. Also after each repair of failed unit, the repaired unit should be sent for final testing to check whether the repaired unit is perfect or not, before sending it for operation. If it is found to be imperfect than replace it by the new one.

Keeping the above view, we in the present chapter provides the reliability analysis of a 2-out-of-3 unit system of cylinders in an automobile engine by incorporating the idea of final testing before use for the repaired failed cylinder.

Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained.

(1) Transition and steady state transition probabilities
(2) Mean Sojourn times in various states
(3) Mean time to system failure

II. MODEL DESCRIPTION AND ASSUMPTIONS

A. The system of automobile engine consists of three identical cylinders. Initially all the three cylinders are operative and each of the unit has only two modes i.e. Normal and Complete failure.

B. The system of automobile engine may operates satisfactorily if at least 2 out of 3 cylinders are in operative mode but with the increased failure rate of the remaining cylinders.

C. A single repair facility with discipline "FCFS" is considered.

D. After each repair of failed cylinder it is sent for final testing where the repaired cylinder will be operative for a fixed period of time. If it works satisfactorily up to a fixed amount of time in testing then it goes for operation otherwise replace it by the new one. The probability that the repaired cylinder will found to be satisfactorily in testing is fixed.

E. The failure time distribution of operative cylinder is exponential while the distribution of completing repair,final testing and replacement are general.

III. NOTATION AND SYMBOLS

N_o : Normal cylinder kept as operative
F_r : Failed cylinder under repair
F_{wr} : Failed cylinder waiting for repair
R_{ft} : Repaired cylinder under final testing
RFT : Final testing of repaired cylinder is continued from earlier state
F_{rep} : Failed cylinder under replacement
F_R : Repair of a failed cylinder is continued from earlier state
FREP: Replacement is continued from earlier state

α : Constant failure rate of operative cylinder
$\beta(>\alpha)$: Constant failure rate of operative unit when one of the unit has already failed
p(=1-q) : Probability that the repaired cylinder works satisfactorily in final testing
g(.), G(.) : pdf and cdf of repair time distribution of the failed cylinder
h(.), H(.) : pdf and cdf of completing final trial of the repaired cylinder
k(.), K(.) : pdf and cdf of replacement time
m_1, m_2, m_3: Mean time for repair, Final testing and replacement

Using the above notation and symbols the possible states of the system are

Up States

$S_o \equiv$ (NO, NO, NO) $S_1 \equiv$ (NO, NO, Fr)
$S_3 \equiv$ (NO, NO, Nft) $S_5 \equiv$ (NO, NO, Frep)

Down States

$S_2 \equiv$ (NO, Fwr, FR) $S_4 \equiv$ (NO, Fwr, Nft)
$S_6 \equiv$ (NO, Fwr, NFT) $S_7 \equiv$ (NO, Fwr, Frep)
$S_8 \equiv$ (NO, Fwr, FREP)

The states S_2, S_6 and S_8 are non-regenerative states and the transition between the various states are shown in Fig. 1.

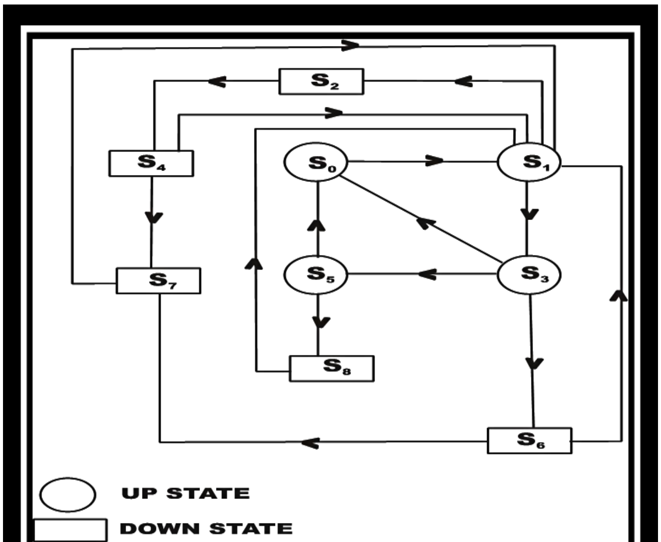

Fig 1

IV. TRANSITION PROBABILITIES

Let T_0 (=0), T_1, T_2,.... be the epochs at which the system enters the states Si $\in$ E. Let Xn denotes the state entered at epoch T_{n+1} i.e. just after the transition of Tn. Then {Tn,Xn} constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = \Pr[X_{n+1} = S_k, T_{n+1} - T_n \leq t \mid X_n = S_i] \quad ....(1)$$

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty) \quad ....(2)$$

Thus one gets

$$p_{01} = p_{71} = 1$$

$$p_{12} = p^{(2)}_{14} = [1-g^*(2\beta)]$$

$$p_{13} = g^*(2\beta)$$

$$p_{30} = p.h^*(2\beta)$$

$$p_{35} = q.h^*(2\beta)$$

$$p^{(6)}_{31} = p[1-h^*(2\beta)]$$

$$p^{(6)}_{37} = q[1-h^*(2\beta)]$$

$$p_{41} = p$$

$$p_{47} = q$$

$$p_{50} = k^*(2\beta)$$

$$p_{58} = [1 - k^*(2\beta)] = p^{(8)}_{51} \quad ....(3\text{-}13)$$

From the above probabilities the following relations can be easily verifies as;

$$p_{01} = 1 = p_{71}$$

$$p_{12} + p_{13} = 1 = p_{13} + p^{(2)}_{14}$$

$$p_{30} + p_{35} + p_{36} = 1 = p_{30} + p_{35} + p_{36} + p^{(6)}_{31} + p^{(6)}_{37}$$

$$p_{41} + p_{47} = 1 = p_{50} + p_{58} = p_{50} + p^{(8)}_{51} \quad ....(14\text{-}17)$$

V. MEAN SOJOURN TIMES

The mean time taken by the system in a particular state Si before transiting to any other state is known as mean sojourn time and is defined as

$$\mu_i = \int_0^\infty P[T>t]\, dt \quad ....(18)$$

Where T is the time of stay in state Si by the system.

To calculate mean sojourn time μ_I in state Si, we assume that so long as the system is in state Si, it will not transit to any other state. Therefore;

$$\mu_0 = \frac{1}{3\alpha}$$

$$\mu_1 = \frac{1}{2\beta}\left[1 - g^*(2\beta)\right]$$

$$\mu_2 = \int_0^\infty \overline{G}(t)\,dt = \int_0^\infty t.g(t)\,.dt$$

$$\mu_3 = \frac{1}{2\beta}\left[1 - h*(2\beta)\right]$$

$$\mu_4 = \int_0^\infty \overline{H}(t)\,dt = \int_0^\infty t.h(t)\,.dt = \mu_6$$

$$\mu_5 = \frac{1}{2\beta}\left[1 - k*(2\beta)\right]$$

$$\mu_7 = \int_0^\infty \overline{K}(t)\,dt = \int_0^\infty t.k(t)\,.dt = \mu_8 \qquad \text{....(19-25)}$$

The unconditional mean time taken by the system to transit to any regenerative state $S_i \in E$, when it (time) is counted from the epoch of entrance into state $Si \in E$, mathematically

$$m_{ij} = -\int_0^\infty t.dQ_{ij}(t) \qquad \text{....(26)}$$

Therefore we get

$m_{01} = \mu_0$

$m_{12} + m_{13} = \mu_1$

$m_{13} + m^{(2)}_{14} = \int_0^\infty t.g(t)\,dt = m_1 = m_{24}$

$m_{30} + m_{35} + m_{36} = \mu_3$

$m_{50} + m_{58} = \mu_5$

$m_{30} + m_{35} + m^{(6)}_{31} + m^{(6)}_{37} = h(t)\,dt = m_2 = m_{41} + m_{47}$

$m_{50} + m_{51} = k(t)\,dt = m_3 = m_{71} = m_{81}$ ....(27-33)

VI. MEAN TIME TO SYSTEM FAILURE (MTSF)

To obtain the distribution function $\pi_i(t)$ of the time to system failure with starting state S0.

$\pi_0(t) = Q_{01}(t)\$\pi_1(t)$

$\pi_1(t) = Q_{13}(t)\$\pi_3(t) + Q^{(2)}_{14}(t)$

$\pi_3(t) = Q_{30}(t)\$\pi_0(t) + Q_{35}(t)\$\pi_5(t) + Q^{(6)}_{31}(t)\$\pi_1(t) + Q^{(6)}_{37}(t)$

$\pi_5(t) = Q_{50}(t)\$\pi_0(t) + Q^{(8)}_{51}(t)\$\pi_1(t)$ ....(34-37)

Taking Laplace Stieltjes transform of relations (34-37) and solving them for 0(s) by omitting the argument 's' for brevity one gets,

$$\tilde{\pi}_0(s) = N_1(s)/D_1(s) \qquad \text{....(38)}$$

where

$$N_1(s) = \tilde{Q}_{01}\tilde{Q}^{(2)}_{14} + \tilde{Q}_{01}\tilde{Q}_{13}\tilde{Q}^{(6)}_{37} \qquad \text{...(39)}$$

and

$$D_1(s) = 1 - \tilde{Q}_{13}(\tilde{Q}_{35}\tilde{Q}^{(8)}_{51} + \tilde{Q}^{(6)}_{31}) - \tilde{Q}_{01}\tilde{Q}_{13}(\tilde{Q}_{30} + \tilde{Q}_{35}\tilde{Q}_{50}) \qquad \text{....(40)}$$

Therefore, mean time to system failure when the initial state is S0, is

$$E(T) = \frac{d}{ds}\pi_0(s)\big|_{s=0} = \frac{D'_1(0) - N'_1(0)}{D_1(0)} = N_1/D_1 \qquad \text{....(41)}$$

where N1 and D1 are same as

$N_1 = \mu_0(1 - p_{13}p^{(6)}_{31} - p_{13}p_{35}p^{(6)}_{51}) + m_1 + m_2p_{13} + m_3p_{13}p_{35}$ ----(42)

and

$D_1 = 1 - p_{13} + p_{13}p^{(6)}_{37}$ ....(43)

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