International Journal of Technical Research and Applications e-ISSN: 2320-8163,
www.ijtra.com Volume3, Issue 2 (Mar-Apr 2015), PP. 272-277

Profit Optimization in A Two-Unit Maintained Standby Redundant System

Dr. Sushant pandey
Dept Of management ,Ishan insititute of mafagenment & technology
Gr. Noida

***Abstract*— Die Arbeit beschaftigt sich mit einem doublierten System bei kalter Reserve emit ex oven verteilter Ausfallzeit and be-Profiliebigen Reparaturzeitverteilungen. Profitgleichungen warden unto- allgemeinsten Kostenstrukturen abgeleitet, wobei die trate des Systems in einem Zustand von drei Parametern abhangt — dem gegenwartigen Zustand, dem nachsten, .1 nspektionszustand und der Zeit, zu der die Profitrate gemessen wird. Die L a pl a c e - Transformierten des Profits, den das System in einer gegebenen Zeit abwirft, we die Diskontrate exponentiell ist, werden erhalten. Das asymptotische erhallen des Profits wird ebenfalls diskutiert. Weiterhin wird der Profit als fir die Effektivitat des Systems vorge-schlagen. Ffir den Fall, daft in jedem Zustand zwischen verschiedenen Moglichkeiten der Steuerung zu entscheiden ist, wird ein Optimierungsproblem zur Bestimmung der optimalen Steuerungsstrategie formuliert, welches den Langzeitprofit maxi-miert. ilowa rds Iterationsmethode wird benutzt, um einen Algorithmus zur Bestimmung der optimalen Strategie (fur das System zu entwickelne The paper deals with a 2-unit cold standby redundant system with exponential failure-time and general repair-time dis-tributions. Profit equations for the system have been developed under the most generalized cost structure in which the earning rate of the system in a state depends upon three parameters — present state, future state of visit and time at which the earning rate is measured. Laplace transforms of the profit that the system will earn in a given time when the discounting rate is ex-ponential, are obtained. Limiting behaviour of the profit has also been discussed. Further, profit has been suggested as the measure of system's effectiveness. When different alternatives are available to a decision maker in each state to operate the system, an optimization problem to determine an optimal operating policy for the system which maximizes long term profit has been formulated. Howard's policy iteration method is used to develop an algorithm to determine an optimal policy for the system.**

Keywords: Communications, Construction Partisipatori, Diffusion of innovation Adoption, Self-help Housing Stimulants Help.

I. Introduction

It is the complexity and multiplicity of influential factors which make the operational environment of standby redundant systems quite sophisticated and involved. Some of the factors are critical as they have direct and signi-ficant impact upon system's performance whereas there are factors that have marginal contribution in generating environment under which systems have to operate. In order to have better understanding of a standby system its operational conditions and the interactions between the two, economic analysis is of paramount importance. More precisely cost feasibility and cost sensitivity are two major measures in any evaluation study. Unfortunately, economics of standby redundant systems has •not received sufficient attention of workers in this area. Of course, some of the papers did concentrate on economic aspects of standby systems (MINE and KAwm. [9], KUMAR [6]), but they are not significant when compared with the number of articles appeared to describe the stochastic behaviour of such systems by obtaining mean-time-to-system failure, steady-state availability, mean recurrence time to a state etc. (BRANsoN and SHAH [1], NAKAGAWA and OSAKI Pit KUMAR [7], GOPALAN and D'SOUZA [3], KISTNER and SUBRA1VIANIAN [5], CHOW [2]). For a 2-unit parallel redundant system with good, degraded and failed states a main-tenance policy was discussed that maximizes the net expected profit rate from the system over an infinite time span (MINE and KAWAI [9]). Later, they considered inspection and replacement policy for one unit system (MINE and KAWAJ [10]).

As expected profit is one of the most important parameters in economic evaluation of standby redundant systems, a few papers have appeared to obtain analytic expressions for expected profit in a standby system ope-rating under different operational environment (KUMAR [6], KUMAR [7], KUMAR and LAL [8]). In all the papers, a simple cost structure is considered viz.,

a) fixed earning (loosing) rate of the system in each state;
b) fixed rewards (costs) at the time of transitions;
c) no discounting of payments received in future.

In a recent paper KUMAR and LAL [8]) the authors have given a hint to relax c) only. However, the other two assumptions a) and b) also need be relaxed as situations do arise when the earning rate of a system depends not only on the present state but also depends upon the future state of visit and it is not fixed throughout the duration of stay in a state. Further, in the above paper two optimization problems were posed for future work.

The purpose of the present paper is two-fold:

(1) to present the most generalized cost model and discuss profit .i.e. to. relax a), b) and c)) simultaneously;
(2) to present a solution procedure to optimization) problem 2 given in KUMAR and LAL [8].
This problem to be determine optimal maintenance policy (maximized profit) in each state for a standby system.

As our objective is not in the direction of complicating system configuration, but it is to show the feasibility of profit evaluation of a standby system under the most generalized cost structure, we take a usual 2-unit cold standby system for the purpose of analysis. We divide the paper in the following two parts :

Part I: Profit of the system.
Part II: Optimal maintenance policy for the system.

In part I: we superimpose the most generalized reward structure (HowARD [4]) on the standby system and develop profit equations under the generalized set-up. The equations are solved for a particular case when the earning rate in a state is a function of the present state and the future state of visit and is independent of time and transition rewards are also constant. Part II deals with determining an optimal maintenance policy for the system in each state that maximizes the expected profit rate of the system. We again make use of HOWARD'S Policy Iteration [4].

Part 1: Profit of the system

The standby system
1. There is a 2-unit cold standby redundant system ; units are identical.
2. The failure-time distribution of the operative unit is exponential with rate 2 and the repair-time is general, say g(t).
3. The system states and transitions between them are

S_o: One unit is operative, the other is as standby; (can go to S_1)
S_i : One unit is under repair, the other is operative; (can go to S_o or S_2).
S_2: One unit is under repair, the other waits for repair; (can go to S_i).

The system is up in S_o — S_i, and it is down in S_2. The process generated by the system model is semi-lVIARKov (BRANs0N and SHAH Pi, KUMAR [6, 7]).

II. THE GENERALIZED REWARD structure

a. While the system occupies Si having chosen a successor state Si, it earns reward at a rate yid (a') at a time after entering Si. This is the yield rate of Si at time a when the successor state is S_i.
b. When the transition from Si to Si is actually made at some time -t, the process earns a bonus bii(-r), a fixed sum.
c. The discounting rate is exponential with rate a, i.e. a unit sum of money at time t in the future has a worth or present value e-at today, a≥0.

III. NOTATIONS

$v_i(t, \alpha)$: the expected present value of the reward the process will generate in a time interval of length t if it is placed in Si at the beginning of this interval.

$v_i(0)$: the additional fixed payment if the system occupies Sj at the end of the interval taken in v2 (t, α).

p_{ij}: the one-step transition probability from S_i to S_1.

$h_{ij}(t)$: the holding-time probability distribution function of the system in Si before making a transition to Si.

$$h_{ij}(t) = \sum_j p_{ij} h_j(t).$$

f*(s) denotes the LA_PLACE transform of a function evaluated at s, e.g. $f^*(s) = \int_0^\infty e^{-st} f(t)dt$.

f ... implies f unless stated otherwise.

- denotes the complement, e.g. fit) = 1 — f(t)

$H_{ij}(i)$: the capital letters in general stand for the continuous distribution function of the corresponding lower case
e.g. $H_{ij}(t) = \int_0^\tau h_{ij}(t)dt$.

IV. SYSTEM PROFIT EQUATIONS

It has been shown in HOWARD [4] that for any state S_i,

$$v_i(t,\alpha) = y_i(t,\alpha) + e^{-\alpha t} v_i(0)\, \bar{H}_i(t) + r_i(t,\alpha) + \sum_j p_{ij} \int_0^t h_{ij}(\tau)\, e^{-\alpha\tau}\, v_j((t-\tau),\alpha)\, d\tau ,$$

where

$$y_i(t,\alpha) = \sum_j p_{ij} y_{ij}(t,\alpha)\, H_{ij}(t) ,$$

$$r_i(t,\alpha) = \sum_j p_{ij} \int_0^t h_{ij}(\tau) \left[\int_0^\tau e^{-\alpha\sigma}\, y_{ij}(\sigma)\, d\sigma + e^{-\alpha\tau}\, b_{ij}(\tau) \right] d\tau$$

Taking the LAPLACE transform of (1)—(3) and writing in matrix notation one obtains

$$\underline{v}^*(s,\alpha) = [\underline{\underline{I}} - \underline{\underline{P}} \,\square\, \underline{\underline{H}}(s+\alpha)]^{-1} [\underline{Y}^*(s,\alpha) + \underline{\underline{W}}^*(s+\alpha)\,\underline{V}(0) + \underline{r}^*(s,\alpha)]$$

where double bar below a letter stands for matrix and single bar (—) denotes vector and r] denotes box opera. tion,

e.g. if

$A=((a_{ij}))$, $B=((b_{ij}))$,

$$C = A \,\square\, B \Rightarrow c_{ij} = a_{ij} b_{ij} \quad \text{if} \quad C = ((c_{ij})) .$$

In order to write profit equations for the model under consideration, let us write there quired parameters fo.r, the model. The following parameters can be obtained directly from BRANSON and SHAH [1] or KUMAR [6, 7] as this model is a special case of these models.

$$p_{01} = 1, \quad p_{10} = \int e^{-\lambda t} g(t)\, dt = g^*(\lambda), \quad p_{12} = \int \lambda e^{-\lambda t} \bar{G}(t)\, dt = \bar{g}^*(\lambda), \quad p_{21} = 1,$$

$$h_{01}(t) = \lambda e^{-\lambda t}, \quad h_{10}(t) = \frac{e^{-\lambda t} g(t)}{g^*(\lambda)}, \quad h_{12}(t) = \frac{\lambda e^{-\lambda t} \bar{G}(t)}{\bar{g}^*(\lambda)}, \quad h_{21}(y) = \frac{\lambda e^{\lambda y}}{\bar{g}^*(\lambda)} \int_y^\infty e^{-\lambda t} g(t)\, dt,$$

$$h_0(t) = h_{01}(t), \quad h_1(t) = e^{-\lambda t} g(t) + \lambda e^{-\lambda t} \bar{G}(t), \quad h_2(t) = h_{21}(t),$$

where y is the remaining repair time of the unit which was under repair at the instant when 82 is entered. Substituting the required values into (2)—(3) from (5), one gets

$$y_0(t,\alpha) = y_{01}(t,\alpha)\, e^{-\lambda t},$$

$$y_1(t,\alpha) = y_{10}(t,\alpha) \int_t^\infty e^{-\lambda\tau} g(\tau)\, d\tau + y_{12}(t,\alpha)\, \lambda \int_t^\infty e^{-\lambda\tau} \bar{G}(\tau)\, d\tau,$$

$$y_2(t,\alpha) = \frac{y_{21}(t,\alpha)}{\bar{g}^*(\lambda)} \int_t^\infty \lambda e^{\lambda y} \left(\int_y^\infty e^{-\lambda z} g(z)\, dz \right) dy,$$

$$r_0(t,\alpha) = \lambda \int_0^t e^{-\lambda\tau} \left[\int_0^\tau e^{-\alpha\sigma} y_{01}(\sigma)\, d\sigma + e^{-\alpha\tau} b_{01}(\tau) \right] d\tau$$

$$r_1(t,\alpha) = \int_0^t e^{-\lambda t} g(t) \left[\int_0^\tau e^{-\alpha\sigma} y_{10}(\sigma)\, d\sigma + e^{-\alpha\tau} b_{10}(\tau) \right] d\tau + \lambda \int_0^t e^{-\lambda t} \bar{G}(t) \left[\int_0^\tau e^{-\alpha\sigma} y_{12}(\sigma)\, d\sigma + e^{-\alpha\tau} b_{12}(\tau \right.$$

$$r_2(t,\alpha) = \int_0^t \frac{\lambda e^{\lambda y}}{\bar{g}^*(\lambda)} \left(\int_y^\infty e^{-\lambda z} g(z)\, dz \right) \left[\int_0^y e^{-\alpha\sigma} y_{21}(\sigma)\, d\sigma + e^{-\alpha\tau} b_{21}(\tau) \right] dy$$

$$[\underline{\underline{I}} - \underline{\underline{P}} - \underline{\underline{H}}^*(s+\alpha)]^{-1} = \frac{1}{1-ab-cd} \begin{bmatrix} 1-cd & a & ac \\ b & 1 & c \\ bd & d & 1-ab \end{bmatrix}$$

Where,

$$a = \lambda/(\lambda+\alpha), \quad b = g^*(\lambda+\alpha), \quad c = \lambda \bar{g}^*(\lambda+\alpha)/(\lambda+\alpha)$$

$$d = \lambda(g^*(\alpha) - g^*(\lambda))/(\lambda-\alpha)\, \bar{g}^*(\lambda) .$$

Now all thequantities required in (1) or (4) are available. They can be substituted and the discounted profit can be obtained when the system starts in any of the states So, Si and S2. We below consider a particular case and discuss in detail profit evaluation.

V. A PARTICULAR REWARD STRUCTURE

To illustrate the computation procedure let us consider the case of constant yield rates and bonuses as below

Yij(σ)=yij, bij(τ)=bij.

Then

$$y_{ij}(\tau,\alpha) = \int_0^\tau e^{-\alpha\sigma} y_{ij}\, d\sigma = \begin{cases} y_{ij} \dfrac{(1-e^{-\alpha\tau})}{\alpha} & \text{if } \alpha > 0 \\ y_{ij}\tau & \text{if } \alpha = 0 . \end{cases}$$

S_0, (6)—(11) reduce to

$$y_0^*(s,\alpha) = y_{01}/(\lambda+s)\,(\alpha+\lambda+s),$$

$$y_1^*(s,\alpha) = \frac{y_{10}}{\alpha s(\alpha+s)} [(\alpha+s)(g^*(\lambda) - g^*(\lambda+s)) - s(g^*(\lambda) - g^*(\alpha+\lambda+s))] +$$

$$+ \frac{y_{12}}{\alpha(\lambda+s)\,(\alpha+s)\,(\alpha+\lambda+s)\,\bar{g}^*(\lambda)} [(\alpha+s)(\alpha+\lambda+s)\{(\lambda+s)\,\bar{g}^*(\lambda) - \lambda\bar{g}^*($$

$$- (\lambda+s)\{(\alpha+\lambda+s)\,\bar{g}^*(\lambda) - \lambda\bar{g}^*(\alpha$$

$$y_2^*(s,\alpha) = \frac{y_{21}}{\alpha}(L^*(s) - L^*(s+\alpha)),$$

$$L^*(s) = \frac{1}{s}(g^*(\lambda) - g^*(2\lambda) + g^*(s) - g^*(s-\lambda)),$$

$$r_0^*(s,\alpha) = \frac{\lambda}{s(\lambda+s)\,(\alpha+\lambda+s)} (y_{01} + b_{01}(\lambda+s)/\alpha),$$

$$r_1^*(s,\alpha) = \frac{1}{\alpha s} \left[y_{10}(g^*(\lambda+s) - g^*(\alpha+\lambda+s)) + \lambda y_{12} \left(\frac{\bar{g}^*(\lambda+s)}{\lambda+s} - \frac{\bar{g}^*(\alpha+\lambda+s)}{\alpha+\lambda+s} \right) \right] +$$

$$+ \frac{b_{10}}{s} g^*(\alpha+\lambda+s) + \frac{\lambda b_{12} \bar{g}^*(\alpha+\lambda+s)}{s(\alpha+\lambda+s)},$$

$$r_2^*(s,\alpha) = \frac{\lambda y_{21}}{s\alpha\bar{g}^*(\lambda)} \left[\frac{g^*(\lambda) - g^*(s)}{s-\lambda} - \frac{g^*(\lambda) - g^*(\alpha+s)}{s-\lambda+\alpha} \right] +$$

$$+ \frac{\lambda b_{21}}{(s-\lambda+\alpha)\,\bar{g}^*(\lambda)} (g^*(\lambda) - g^*(\alpha+s)) .$$

Substituting of (15) —(20) into (4), one gets the elements of the expected profit vector v*(8, a) as below

$$v_0^*(s,\alpha) = \frac{1}{1-ab-cd} \left[(1-cd) \left\{ y^*(s,\alpha) + \sum_j H_{0j}^* v(\alpha+s)\, v_0(0) + r_0^*(s,\alpha) \right\} + \right.$$

$$\left. + a \left\{ y_1^*(s,\alpha) + \sum_j H_{1j}^*(\alpha+s)\, v_j(0) + r_1^*(\alpha,s) \right\} \right],$$

$$v_1^*(s,\alpha) = \frac{1}{1-ab-cd} \left[b \left\{ y_0^*(s,\alpha) + \sum_j H_{0j}^*(\alpha+s)\, v_j(0) + r_0^*(s,\alpha) \right\} + \right.$$

$$+ y_1^*(s,\alpha) + \sum_j H_{1j}^*(\alpha+s)\, v_j(0) + r_1^*(s,\alpha) + c \left\{ y_2^*(s,\alpha) + \sum_j H_{2j}^*(\alpha+s)\, v_j(0) + r_2^*(\right.$$

$$v_2^*(s,\alpha) = \frac{1}{1-ab-cd} \left[bd \left\{ y_0^*(s,\alpha) + \sum_j H_{0j}^*(\alpha+s)\, v_j(0) + r_0^*(s,\alpha) \right\} + \right.$$

$$+ d \left\{ y_1^*(s,\alpha) + \sum_j H_{1j}^*(s+\alpha)\, v_j(0) + r_1^*(s,\alpha) \right\} +$$

$$\left. + (1-ab) \left\{ y_2^*(s,\alpha) + \sum_j H_{2j}^*(s+\alpha)\, v_j(0) + r_2(s,\alpha) \right\} \right].$$

Limiting behaviour of expected profit From the final value theorem one knows

$$v_i(\alpha) = \lim_{t\to\infty} v_i(t,\alpha) = \lim_{s\to 0} s\, v_i^*(s,\alpha) \quad \text{for all} \quad i = 0, 1, 2.$$

Then it easily follows that

$$\underline{v}^*(\alpha) = [\underline{\underline{I}} - \underline{\underline{P}} \square \underline{\underline{H}}^*(\alpha)]^{-1} \underline{r}^*(\alpha)$$

$$\underline{v}^*(\alpha) = \underline{r}^*(\alpha) + \underline{\underline{P}} \square \underline{\underline{H}}^*(\alpha) \underline{v}^*(\alpha) .$$

It implies the set of simultaneous equations

$$v_i^*(\alpha) = r_i^*(\alpha) + \sum_j p_{ij} h_{ij}^*(\alpha) v_j^*(\alpha)$$

$$r_i(\alpha) = \sum_j p_{ij} \int_0^\infty h_{ij}(\tau) \left[\int_0^\tau e^{-\alpha\sigma} y_{ij}(\sigma) \, d\sigma + e^{-\alpha\tau} b_{ij}(\tau) \right] d\tau .$$

For the particular case when yield rate and bonuses are constant, we get

$$r_i^*(\alpha) = \sum_j p_{ij} y_{ij} \frac{h_{ij}^*(\alpha)}{\alpha} + \sum_i p_{ij} b_{ij} h_{ij}^*(\alpha) .$$

Therefore, if one is interested in long term profit viz., the expected present values vi(a) of the entering state Si iii a system that will continue indefinitely, the following steps are straight forward :

a. Define the system states. Identify the system as up or down in each state. State transitions between dif-ferent states.
b. write transition probabilities, holding time probability distribution functions, yield rates along with transition rewards and discounting rate. Compute LAPLACE transforms of holding time probability distributia functions evaluated at the point s = a, the discounting rate.
c. Compute ri(A) using (28) or (29) for each state Si and substitute in (27) to get a set of simultaneous 729 tic tions describing profit. The number of equations is equal to the number of states.

For the particular case under discussion we have

$$v_1^*(\alpha) = \frac{r_1^*(\alpha) + h_{10}^*(\alpha) r_0^*(\alpha) + h_{12}^*(\alpha) r_2^*(\alpha)}{1 - h_{12}^*(\alpha) h_{21}^*(\alpha) - h_{01}^*(\alpha) h_{10}^*(\alpha)},$$

$$v_0^*(\alpha) = r_0^*(\alpha) + h_{01}^*(\alpha) v_1^*(\alpha), \qquad v_2^*(\alpha) = r_2^*(\alpha) + h_{21}^*(\alpha) v_1^*(\alpha),$$

where

$$r_0^*(\alpha) = \left(\frac{y_{01}}{\alpha} + b_{01}\right) \frac{\lambda}{\lambda + \alpha},$$

$$r_1^*(\alpha) = \left(\frac{y_{10}}{\alpha} + b_{10}\right) g^*(\lambda + \alpha) + \left(\frac{y_{12}}{\alpha} + b_{12}\right) \lambda \frac{\bar{g}^*(\lambda + \alpha)}{\lambda + \alpha},$$

$$r_2^*(\alpha) = \left(\frac{y_{21}}{\alpha} + b_{21}\right) \lambda \left(\frac{g^*(\alpha) - g^*(\lambda)}{\bar{g}^*(\lambda) \cdot (\lambda - \alpha)}\right).$$

Part, 11: Optimal maintenance policy for the system

In the maintenance of equipments, one is often faced with the problem of choosing an optimal alternative ernative from a given set of alternatives. More elaborately, to maintain a system, there may be several maintenance schemes available to a decision maker e.g. ordinary maintenance (OM), costly maintenance (CM) and highly expensive main-tenance (HEM). Last policy viz. HEM may involve large expenditure but ensures a higher value for operating time of the system or mean-time-to-system failure, as compared to other policies. One may be interested in selecting the policy (one alternative from each state) that maximizes the expected profit in long run.

In this part we give a general formulation for a semi-MARKov decision process applicable to any standby redundant system. To determine the optimal operating (or maintenance) policy for the system we apply HOWARD's Policy Iteration [4]. To get an insight into the formulation aspect of maintenance policies let us concentrate on the following discussion:

When a standby redundant system is operating, there may be several options available; OM, CM and HEM etc. While a unit of the system is under repair, there may again be different alternatives viz., ordinary repair (OR), costly repair (CR), highly expensive repair (HER) etc. In general in each state of the standby system, there may be several operating alternatives available to a decision maker and the problem is to choose one alternative in each state keeping in view some effectiveness criterion viz., choose the alternative that maximizes long term profit rate of the system or minimizes long term cost rate of the system. Before presenting formal description, we state the following assumptions:

a) The earning rate of the system in each state is constant, that is, it neither depends on the future state of visit nor on time and.

b) whenever, the system changes its state, fixed transition rewards are involved;

c) there is no discounting.

VI. SEMI-MARKOV DECISION PROCESS

Suppose when the system is in S i, there are various alternatives for its operation. Associated with each alternative Su say k, in Si, there are process parameters: transition probabilities (29,.i), holding-time probability distribution func-tions (./11,)(t), earning rate (or loosing rate) (y,) and transition rewards (or costs) (r,16). We assume a finite but different number of alternatives in each state. The problem is to choose one alternative in each state which may be called_ a decision and the set of decisions (one in each state) may be called a policy. We have to find the policy that maxi-inizc8 the, average profit of the system in steady-state.

VII. NOTATIONS

i : subscripts to denote system states : 1, 2, , n

International Journal of Technical Research and Applications e-ISSN: 2320-8163,
www.ijtra.com Volume3, Issue 2 (Mar-Apr 2015), PP. 272-277

vi(t) : total profit we expect the system to earn in time t, if the system starts in Si at time t = 0;
yi : earning rate of the system in Si when alternative k is selected ;
rij ; the system goes from Si to SI and in Si, alternative fixed transition reward when the system goes from si to sj and in si , alternateive k was selected ;
g : expected profit of the system per unit time in steady-state.
Then following HowAith [4] we know that for large t,

$$V_i(t) = v_i + gt$$

where v, is the transient part 0/ the profit and y is the steady-state part, and,

where

$$v_i + g\mu_i = q_i\mu_i + \sum_j p_{ij}r_j , \qquad i = 1, 2, \ldots, n ,$$

$$q_i\mu_i = \sum_j p_{ij}r_{ij} + y_i\mu_i .$$

The steps involved in the policy iteration procedure are:

step: Define the standby redundant system i.e. its state space and transitions between them. Identify up me rate alternative operating rates in each state. Specify earning rate, repair and down states of the system. 141.4niii cost rate, fixed transit ', ion rewards etc.

Step 2: Compute the transition probabilities and the holding time probability distribution functions,

Step 3: Choose one decision in each state i.e. the present policy for which

$$q_i^k = \frac{1}{\mu_i^k} \sum_j p_{ij}^k \cdot r_{ij}^k + y_i^k$$

is maximum.

Step 4: For the policy in step 3 solve the set of it equations

$$v_i + g\mu_i = q_i\mu_i + \sum_j p_{ij}v_j$$

for all i=1,2,….,n

For v_1, v_2, ……v $_{n-1}$ and g by setting v_n=0 .

Step 5: Using the values of v_1, and obtained at step 4 compute the test quantity

$$q_i^k + \frac{1}{\mu_i^k}\left(\sum_j p_{ij}v_j - v_i\right)$$

for each alternative in each state. Choose the alternative in each state for which the test quantity is maximum. Step 6: Examine if the new policy is different from the initial policy. If yes, go to step 4; otherwise the opti• mal policy is reached.

VIII. Application to a standby system with exponential failure and exponential repair time distributions

Denote by : A the failure rate of the system, 77 the repair rate of the failed unit, 'om' the ordinary maintenance, 'or'the► ordinary repair ; 'cm' the costly maintenance, 'cr' the costly repair.

The following table gives the detailed description of various system parameters:

Decisions in different states:
In S_0, — 1 : om, 2 : cm.
In S_1 — 1 : (om, or), 2 : (om, cr), 3 : (cm, or), 4 : (cm, cr).
In S_2, — 1 : or; 2 : cr.

State S_i	Alter-natives k	Transition Probabilities p_{i0}^k	p_{i1}^k	p_{i2}^k	Rewards r_{i0}^k	r_{i1}^k	r_{i2}^k	Mean holding Time μ_i^k	λ,	η,	y_i^k	Earning Rate q
S_0	1	—	1	—	—	−5.0	—	$1/\lambda$	.05,	—	100	99.75
	2	—	1	—	—	−2.5	—	$1/\lambda$	.01,	—	50	49.975
S_1	1	.67	—	.33	−1.0	—	−2.5	$1/(\lambda+\eta)$	.05,	.1,	75	74.776
	2	.80	—	.20	− .5	—	−2.5	$1/(\lambda+\eta)$	.05,	.2,	50	49.775
	3	.67	—	.33	−1.0	—	−2.5	$1/(\lambda+\eta)$	.01,	.1,	25	24.836
	4	.95	—	.05	− .5	—	−2.5	$1/(\lambda+\eta)$	.01,	.2,	10	00.874
S_2	1	—	1	—	—	−1.0	—	$1/\eta$	—,	.1,	−25	−25.1
	2	—	1	—	—	− .5	—	$1/\eta$	—,	.2,	−50	−50.1

So, the initial policy is $\begin{pmatrix}1\\1\\1\end{pmatrix}$ and policy equations are

$$v_0 + g\mu_0 = q_0\mu_0 + v_1 ,$$
$$v_1 + g\mu_1 = q_1\mu_1 + p_{10}v_0 + p_{12}v_2 ,$$
$$v_2 + g\mu_2 = q_2\mu_2 + v_1 ,$$

or, if $v_2 = 0$

$$v_1 - v_0 - 20g = -1995.00 ,$$
$$v_1 - .67v_0 + 6.67g = 498.76 ,$$
$$v_1 - 10g = 251 ,$$

which gives the solution as

v_o = 1496.00 , v_1 = 1001.0 , v_2=0, g=75.00

First policy iterationwe
Using above values of v_0,v_1,v_2 and g we compute the value of test quantityy (TQ)

$$q_i^k + \frac{1}{\mu_i^k}\left(\sum_j p_{ij}v_j - v_i\right)$$

In each state.

State	Alternative	Value of T.Q.
S_0	1	75
	2	45.025
S_1	1	74.976
	2	98.725
	3	24.986
	4	98.116
S_2	1	75
	2	150.1

So, the improved policy is $\begin{pmatrix}1\\2\\2\end{pmatrix}$ and policy equations are

$$v_1 - v_0 - 20g = -1995.00\ ,$$
$$v_1 - .8v_0 + 4g = 199.1\ ,$$
$$v_1 - 5g = 250.5\ ,$$

and the solution is v_0,=999, v_1, = 666, v_2, = 0, g = 83.10. we observe that value of g has been improved from 75 to 83.10. Again we compute T.Q. values in each state.

State	Alternative	Value of T.Q.
S_0	1	83.10
	2	46.645
S_1	1	75.276
	2	83.075
	3	25.202
	4	66.805
S_2	1	41.5
	2	83.1

So, the improved policy is $\begin{pmatrix}1\\2\\2\end{pmatrix}$ and policy equations are

$$v_1 - v_0 - 20g = -1995.00\ ,$$
$$v_1 - .8v_0 + 4g = 199.1\ ,$$
$$v_1 - 5g = 250.5$$

and the solution is v, = 999, v1 = 666, v2 = 0 and g = 83.10. 1) (This policy is the same as obtained in the previous

iteration, hence the optimal policy is reached and is $\begin{pmatrix}1\\2\\2\end{pmatrix}$

Concluding remarks

We have developed profit equations for a 2-unit standby redundant system under a generalized cost structure. The procedure can be applied to compute profit in any standby redundant system. However, as the size of the state space increases, the equations are quite complex and tedious. Under such circumstances it will be desirable to develope computer programmes for the purpose. Further, the optimal maintenance policy has been discussed for a simple 2-unit standby system for illustration but the procedure could easily be applied to other standby systems with exponentail failure-time and general repair-time distributions. To determine optimal rnaintenance4)olicy, computer algorithms will serve a useful purpose for system designers and maintenance engineers.

REFERENCES

[1] BRANSON, M. H.; HAH, B., Reliability analysis of systems comprised of units with arbitrary repair-time distributions, IEEE S

[2] CHOW, D. K., Availability of some repairable computer systems, IEEE Trans. Rel. R-24, No•1, 6466 (1975) Trans. Rel. R-20, No. 4, 217-223 (1971).

[3] GOPALAN, M. N.; D'SoCrZA, C. A., Probabilistic analysis of a system with two dissimilar units s4ject to preventive maintenance and a single service facility, Operations Research 23, No. 3, 534-548 (1975). 5

[4] HOWARD, R. A., Dynamic Probabilistic Systems. Vol. II, New York, Wiley 1971.

[5] KISTNER, K. P.; StrBRAmANIAN, R., The reliability of a system with redundant repairable components subject to failure, Z. Z Operations Res. 18, 117-129 (1974) (In German).

[6] KUMAR, ASHOK, Profit evaluation in some repairable redundant system, ZAMM 57, 485-489 (19.77).

[7] KISTNER, K. P.; StrBRAmANIAN, R., The reliability of a system with redundant repairable components subject to failure, Z.

[8] KUMAR, ASHOK, Stochastic behaviour of a 2-unit standby redundant system with two switching failure modes, Math. Operations-7 forsch. Statist., Ser. Optimization 8, 571-580 (1977)

[9] KUMAR, ASHOK; LAL, ROSHAN, Stochastic behaviour of a two-unit standby system with contact failure and intermittently available repair facility, Internat. J. System. Sci. 10, 589-603 (1979).

[10] MINE, H. ; KAWAI, H., An optimal maintenance policy for a 2-unit parallel system with degraded states, IEEE Trans. Rel. R-23, 81-85 (1974).

[11] MINE, H. ; KAWAI, H., An optimal inspection and replacement policy, IEEE Trans. Rel. R-24, 305-309 (1975).

[12] NAKAGAWA, T. ; ()SAKI, S., A summary of optimum preventive maintenance policies for a 2-unit standby redundant system, Z. Operations. Res. 22, 171-187 (1976).

[13] Eingereicht am 10. 1. 1980

[14] Anschriften: Dr. ASHOK KUMAR, Training Group, Defence Science Laboratory, Metcalfe House, Delhi-110054, India;, ROSHAN LAL, Research Fellow, Department of Statistics, Institute of Social Sciences, Agra, India..