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MULTISTEP METHOD FOR HIGHER ORDER LINEAR DIFFERENTIAL EQUATION

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***Abstract*— This paper contain the solution of higher order linear differential equation solution by the numerical method using the multistep method for solving these kind of differential equation and compare the result with single step and semi numerical methods.**

***Index Terms*— Multistep method, Milnes method, A.B.M method, RK method ,higher order linear differential equations.**

I. INTRODUCTION

Calculus has provided various method for closed form solution of dy/dx =f(x,y) as y= g(x) +c the value of c can be obtained by the initial solution such as y=y0 when x= x0 . based on our calculations we find that although there are many methods available in the calculus, still there are many first order and higher order differential equations which can't be solved by these methods and we require some other method which can tackle various problem and are computer oriented for as or and y(a)= b have no solution. By the calculus method but they can be solved by the numerical method. Our aim in this paper to solve the problem by numerical method and compare the result. The methods applied for the solution are :

A. Semi numerical method (Picard's method)

B. Single step method by R. K method of fourth order

C. Multi step method (Milnes, A.B.M predictor corrector methods)Type Style and Fonts

II. MULTI STEP PREDICTOR CORRECTOR METHOD

The figure on the right illustrates the (familiar) fact that if you know y ′ (xi), i.e. the slope of y(x), then you can compute a first-order accurate approximation Y 1st order i+1 to the solution yi+1. Likewise, if we know the slope and the curvature of our solution at a given point, we can compute a second-order accurate approximation, Y 2nd order i+1 , to the solution at the next step.
we find approximation to using already computed value $y_{i-k}, k = 0,1,2 \ldots$ now

$$y''_i = \frac{y'_i - y'_{i-1}}{h} = \frac{f_i - f_{i-1}}{h} \qquad (1.1)$$

Here and below we will use the notation fi in two slightly different ways:

$$fi \equiv f(xi, yi) \text{ or } fi \equiv f(xi, Yi) \qquad (1.2)$$

Whenever this does not cause any confusion.
Continuing with Eq. (1.1), we can state it more specifically by writing

$$y''_i = \frac{y'_i - y'_{i-1}}{h} + O(h) = \frac{f_i - f_{i-1}}{h} + O(h) \qquad (1.3)$$

where we will compute the *O*(*h*) term later. For now, we use (1.3) to approximate *yi*+1 as follows:

$$y_{i+1} = y(x_i + h) = y_i + hy'_i + \frac{h^2}{2} y''_i + O(h^3)$$

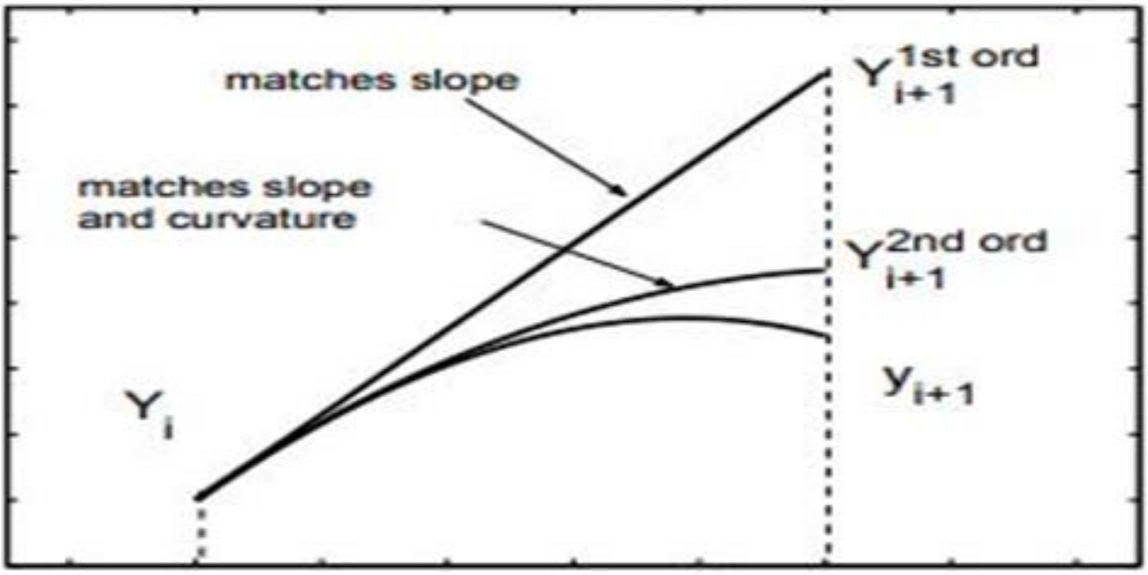
$$= y_i + hf_i + \frac{h^2}{2}\left(\frac{f_i - f_{i-1}}{h} + O(h)\right) + O(h^3)$$

$$= y_i + h\left(\tfrac{3}{2} f_i - \tfrac{1}{2} f_{i-1}\right) + O(h^3) \qquad (1.4)$$

Remark 1: To start the corresponding finite-difference method,

i.e. $$Y_{i+1} = Y_i + h(\tfrac{3}{2} f_i - \tfrac{1}{2} f_{i-1}) \qquad (1.5)$$

(now we use fi as $f(xi, Yi)$), one needs *two* initial points of the solution, $Y0$ and $Y1$.
These can be computed.



Remark 2: Equation (1.4) becomes *exact* rather than approximate if

$y(x) = p2(x) \equiv ax^2 + bx + c$ is a second-degree polynomial in *x*.

Indeed, in such a case

$$y'_i = 2ax_i + b \text{ and } y''_i = (y'_i - y''_{i-1})/h = 2a \quad (1.6)$$

(note the exact equality in the last formula). We will use this remark later on.

Method (1.5) is of the second order. If we want to obtain a third-order method along the same lines, we need to use the third derivative of the solution:

$$y'''_i = \frac{y'_i - 2y'_{i-1} + y'_{i-2}}{h^2} + O(h) \quad (1.7)$$

We will proceed As in Eq. (1.4), namely:

$$y_{i+1} = y_i + hy'_i + \frac{h^2}{2}y''_i + \frac{h^3}{6}y'''_i + O(h^4) \quad (1.8)$$

If now we try to substitute the expression on the r.h.s. of (1.3) for y''_i, we notice that we actually need an expression for the *O*(*h*)-term there that would have accuracy of $O(h^2)$. Here is the corresponding calculation:

$$\frac{(y'_i - y'_{i-1})}{h} = \frac{y'(x_i) - y'(x_{i-1})}{h}$$

$$= \frac{y'_i - [y'_i - hy''_i + \frac{h^2}{2}y'''_i + O(h^3)]}{h} \quad (1.9)$$

$$= y''_i - \frac{h}{2}y'''_i + O(h^2)$$

$$y''_i = \frac{y'_i - y'_{i-1}}{h} + \frac{h}{2}y'''_i + O(h^2) \quad (1.10)$$

To complete the derivation of the third-order finite-difference method, we substitute Eqs. (1.10), (1.7), and $y'i = fi$ etc. into Eq. (1.8).

The result is

$$Y_{i+1} = Y_i + \frac{h}{12}[23f_i - 16f_{i-1} + 5f_{i-2}] \quad (1.11)$$

The local truncation error of this method is $O(h^4)$Method (1.11) is called the 3rd-order **Adams Bashforth method.**

Similarly, one can derive higher-order Adams–Bashforth methods. For example, the 4thorder Adams–Bashforth method is

$$Y_{i+1} = Y_i + \frac{h}{24}[55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3}] \quad (1.12)$$

Methods like (1.5), (1.11), and (1.12) are called *multistep* methods. To start a multistep method, one requires more than one initial point of the solution (in the examples considered above, the number of required initial points equals the order of the method**.**

III. PROBLEM

Let the problem of higher order derivative

$$\frac{d^2y}{dx^2} = x\frac{dy}{dx} + y, y(1) = 1 \; and \; y'(1) = 2 \; find \; y(x = 1.2)$$

Now take $\frac{dy}{dx} = z$, *then the equation chamge to* $\frac{dz}{dx}$= xz + y

Therefore $\frac{dy}{dx} = z, y(1) = 1$

And $\frac{dz}{dx}$= xz + y, z (1)= 2

Therefore $\frac{dy}{dx} = z \; (f_1(x_n, y_n, z_n)$ say and

$$\frac{dz}{dx} = xz + y \quad (f_2(x_n, y_n, z_n))$$

By the Euler's Predictors –Correctors Methods:

$$y_1^P = y_0 + hf(x_0, y_{0,} z_0)$$

Above equations give the Euler's Predictors –Correctors formulae. Predicts the value of In corrector may be repeated several times and we can have several corrections for more accurate result by:

$$y_1^{C(n+1)} = y_0 + \frac{h}{2}\{f(x_0, y_0, z_0) + f(x_1, y_1^{Cn}, z_1^{cn})\}$$

With the initials $x_0 = 1, y_0 = 1, \; z_0 = 2$

$$y_1^P = 1.2 \quad and \; z_1^P = 2.3 \; for \; h = 0.2$$

$$y_1^C = 1.215 \; and \; z_1^C = 2.29$$

For second correction value

$$y_2^P = 1.2145 \quad and \; z_2^P = 2.3367$$

By Milne's Predictors –Correctors Methods:

Milne's Predictors –Correctors method requires four prior values using those data we can find out by formulae:

$$y_4^C = y_2 + \frac{h}{3}[f(x_2, y_2, z_2) + 4f(x_{3,} y_3, z_3) + f(x_4, y_4, z_4)]$$

For

$$\frac{dy}{dx} = z \; say \; f_1(x_0, y_0, z_0)$$

X	1	1.05	1.10	1.15
Y	1	1.1	1.2075	1.32339

$$\frac{dz}{dx} = xz + y \; say \; f_2(x_{0,} y_0, z_0)$$

X	1	1.05	1.10	1.15
Y	2	2.15	2.3178	2.50565

Above tabulated values are calculated by Euler's Method now by the M.P.C Method

$$y_4^P = 1.4662 \; and \; Z_4^P = 2.758264$$

$$y_4^C = 1.439614 \; and \; Z_4^C = 2.740345$$

By Adam-Bashforth's Moulton's Predictors –Correctors Methods:

It give very small error.

$$y_4^P = y_0 + \frac{h}{24}[55f(x_3, y_3, z_3) - 59f(x_2, y_2, z_2) + 37f(x_1, y_1, z_1) - 9f(x_0, y_0, z_0)]$$

$$y_4^C = y_3 + \frac{h}{24}[9f(x_4, y_4, z_4) + 19f(x_3, y_3, z_3) - 5f(x_2, y_2, z_2) + f(x_1, y_1, z_1)]$$

By the above tabulated value calculation of

$$y_4^P = 1.122370 \; and \; Z_4^P = 2.210588$$

$$y_4^C = 1.44435 \; and \; Z_4^C = 2.710734$$

IV. COMPARISON OF MULTISTEP AND SINGLE STEP METHODS

The advantage of multistep over single-step single step methods of the same accuracy is that the multistep methods require only one function evaluation per step, while, e.g., the R.K. method requires 4, and the RK–Feldberg method 6, function evaluations.

V. RESULT

Euler predictor corrector method	$y_2^P = 1.2145$	$z_2^P = 2.3367$
Milne's Predictors –Correctors method	$y_4^C = 1.439614$	$Z_4^C = 2.740345$
Adam-Bashforth's Moulton's Predictors –Correctors Methods	$y_4^C = 1.44435$	$Z_4^C = 2.710734$
Single step method	Y =1.47	Z=2.76

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VI. CONCLUSION

Hence we find multistep method a better approach for solving higher order derivative problem accurate upto more decimal places with less error.

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