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INDEPENDENT AND CONNECTED DOMINATION NUMBER IN MIDDLE AND LINE GRAPHS

B. K. Keerthiga Priyatharsini
Assistant Professor
Department of Mathematics
Anna University
University College of Engineering, Ramanathapuram
Ramanathapuram-623 513
Keerthi.priyatharsini@yahoo.com

Abstract- The middle graph of a graph G, denoted by M(G), is a graph whose vertex set is V(G)UE(G) , and two vertices are adjacent if they are adjacent edges of G or one is a vertex and other is a edge incident with it . The Line graph of G , written L(G), is the simple graph whose vertices are the edges of G, with ef Є E(L(G)) when e and f have a common end vertex in G. A set S of vertices of graph M(G) if S is an independent dominating set of M(G) if S is an independent set and every vertex not in S is adjacent to a vertex in S. The independent middle domination number of G, denoted by iM(G) is the minimum cardinality of an independent dominating set of M(G).A dominating set D is a connected dominating set if <D> is connected. The connected domination number, denoted by $ϒ_c$, is the minimum number of vertices in a connected dominating set. In this paper many bounds on iL(G) ,iM(G),ϒM(G) were obtained in terms of element of G, but not in terms of elements of L(G) or M(G).

Keywords- domination number, Connected domination number, independent domination number, Line graph, Middle graph.

I. INTRODUCTION

The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it because of its many and varied applications in such fields as linear algebra and optimization, design and analysis of communication networks, and social sciences and military surveillance. Many variants of dominating models are available in the existing literature. For a comprehensive bibliography of papers on the concept of domination, readers are referred to Hedetniemi and Laskar. The present paper is focused on connected domination and independent domination in graphs.

In a simple undirected graph G= (V,E) a subset D of V is dominating if every vertex of V-D has atleast one neighbor in D and D is independent if no two vertices of D are adjacent. A set is independent dominating if it is independent and dominating. Let ϒ(G) be the minimum cardinality of a dominating set and let i(G) denotes the minimum cardinality of an independent dominating set of G. A dominating set D is a connected dominating set if <D> is connected. The domination number, denoted by $ϒ_c$ is the minimum number of vertices in a connected dominating set. A Line graph L(G) is a graph whose vertices corresponds to the edges of G and two vertices in L(G) are adjacent if and only if the corresponding edges in G are adjacent. A subdivision of an edge e=uv of a graph G is the replacement of the edge e by a path (u , v , w).The graph obtained from a graph G by subdividing each edge of G exactly once is called the subdivision graph of G and is denoted by S(G).The middle graph of a connected graph G denoted by M(G) is the graph whose vertex set is V(G)UE(G) where two vertices are adjacent if (i) they are adjacent edges of G, or (ii) one is a vertex of G and the other is an edge incident with it .

For any real number n, $\lceil n \rceil$ denotes the smallest integer not less than n and $\lfloor n \rfloor$ denotes the greatest integer not greater than n.

Theorem:1

If every support vertex of tree is adjacent to atleast one end edge , then $i(L(T)) \leq \left\lceil \frac{q-m}{2} \right\rceil + 1$, where m is the number of end edges in T. Equality holds for star $k_{1,p-1}$.

Proof:

Let F={ e_1 , e_2 , …..e_n} be the set of all end edges in T such that │F │ = m. Now without loss of generality, since VLT= ET , let S= F'U H ,where F' ,subset of ,F and ,H ,subset of V(L(T))-F , such that H does not belongs to N[F] be the minimal set of vertices which covers all the vertices LT.

Clearly set of the vertices of a subgraph <S> is independent, then by the above argument S is a minimal independent dominating set of LT. Clearly it follows that, $|S| \leq \left\lceil \frac{q-m}{2} \right\rceil + 1$

Therefore, $i(L(T)) \leq \left\lceil \frac{q-m}{2} \right\rceil + 1$.

Theorem2: For any connected p, q –graph G, $i(L(G)) \leq q - \Delta'(G)$.

Proof: Suppose C={v_1 ,v_2 ,v_3,……,v_k} be the set of all non end vertices in G. Then there exists atleast one vertex v Є C which is incident with atleast one edge e Є Δ'(G) in G.Now without loss of generality in LG, suppose H= { v_1 ,v_2 ,v_3,……,v_n} be the set of all end vertices in LG and if VLG-H=I.

Then there exists a subset D, subset of I, in LG such that the subgraph <D> is independent.

Clearly, D is an i-set of LG. It follows that, $|D| \leq q - \Delta'(G)$ and hence $i(L(G)) \leq q - \Delta'(G)$.

Theorem:3

For any complete bipartite graph $k_{m,n}$, $i_M(K_{m,n})$ =n for n ≥ m.

Proof: Let (X,Y) be a bipartition of $K_{m,n}$,n ≥ m with │X│ = m and │Y│ = n. Let X = {x_1,x_2,x_3,…..x_m} and Y = {y_1,y_2,y_3,….y_n}. Let E_1 = {x_iy_j / 1 ≤ i ≤ m,1 ≤ j ≤ n} be the independent edges in $k_{m,n}$.

Clearly │E_1│ =min(m , n)=m.

In M(G), let $S = \{ v_1, v_2, v_3, \ldots v_k \}$ be the vertices subdividing each edge of G in M(G).

Consider a set $S_1 = \{ v_i / 1 \leq i \leq k \}$, subset of S, be the vertices subdividing the edges of E_1.

Clearly S_1 is an independent set of vertices in M(G).Now let $Y_1=\{y_j / y_j = N(v_i)$,for each v_i belongs to $S_1\}$.

Clearly $|Y_1| = m$.

Without loss of generality, $Y_2=Y-Y_1$ is an independent set of vertices in M(G).

Now, $N(S_1) = XUV(S-S_1)UY_1$ and hence $N[S_1UY_2] = V[M(G)]$.Since $<S_1UY_2>$ is independent, thus the induced subgraph $<S_1UY_2>$ is a minimal independent dominating set in M(G).

Clearly $|S_1|=|E_1| = m$ and $|Y-Y_1| = n-m$ .Therefore $|S_1UY_2| = |S_1| + |Y_2| = m+n-m = n$.

Hence $i_M(k_{m,n}) = n$ where $n \geq m$.

Theorem:4

For complete bipartite graph $k_{m,n}$, $Y_c(M(k_{m,n}))=m+n-1$ for any m ,n.

Proof:

Let (X,Y) be a bipartition of $k_{m,n}$. Let $|X| = m$ and $|Y| = n$

Let $X = \{x_1,x_2,x_3,\ldots x_m\}$ and $Y = \{y_1,y_2,y_3,\ldots y_n\}$

Let $\{ u_{11},u_{12},\ldots u_{ij} , 1 \leq i \leq m , 1 \leq j \leq n \}$ be the added vertices corresponding to the edges $e_{11},e_{12},\ldots e_{ij}$ $1 \leq i \leq m$, $1\leq j \leq n$ of $k_{m,n}$ to obtain $M(k_{m,n})$.

Thus $V(M(k_{m,n}))) =\{ x_1,x_2,x_3,\ldots x_m, y_1,y_2,y_3,\ldots y_n , u_{11},u_{12},\ldots u_{ij} 1 \leq i \leq m , 1 \leq j \leq n \}$

Therefore $|V(M(k_{m,n})))| = mn + m + n$

Consider a set $F =\{ e_{m-i,1} , e_{mj}$ for $i=1,2,3\ldots m-1, j=1,2,3,\ldots n\}$ is a connected dominating set with

$|F| = m + n - 1$.

Since each vertex in $M(k_{m,n})$ is either in F or is adjacent to an vertex in F ,

Therefore, F is connected dominating set. Since, m number of vertices can dominate mn + m + 1 vertices and other vertices can be dominated by n - 1 vertices.

Therefore, any set containing edges less than that of F cannot be connected domainting set of $M(k_{m,n})$.

This implies that F is connected dominating set with minimum cardinality.

Therefore, $Y_c(M(k_{m,n})) = m + n - 1$.

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