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# $\overline{\text{H}}$- FUNCTION AND GENERAL CLASS OF POLYNOMIAL AND HEAT CONDUCTION IN A ROD.

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Abstract - In this paper, first we evaluate a finite integral involving general class of polynomials and the product of two $\overline{\text{H}}$-functions and then we make its application to solve boundary value problem on heat conduction in a rod under the certain conditions and further we establish an expansion formula involving about product of $\overline{\text{H}}$-function. In view of generality of the polynomials and products of $\overline{\text{H}}$-function occurring here in, on specializing the coefficients of polynomials and parameters of the $\overline{\text{H}}$-function, our results would readily reduce to a large number of results involving known class of polynomials and simpler functions.

**Keywords:** General Class of Polynomials, $\overline{\text{H}}$ Function, Jacobi polynomial and Leguerre polynomials.
**Mathematics Subject Classification :** 33C60, 34B05

## I. INTRODUCTION

The general class of polynomials introduced by Shrivastava [7] and defined by [8] and [10] as follows:

$$S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{m,k} A_{n,k}}{k!} x^k, \; n = 0,1,2,...$$
….. (1)

where m is an arbitrary positive integer the coefficient $A_{n,k}$ $(n,k \geq 0)$ are arbitrary constants, real or complex.

$\overline{\text{H}}$-function will be defined and represented as follows [2] and [4]:

$$\overline{H}_{P,Q}^{M,N}[z] = \overline{H}_{P,Q}^{M,N}\left[ z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \bar{\phi}(\xi) z^{\xi} d\xi \; ...(2) \text{ where } \xi \neq 0$$

and

$$\bar{\phi}(\xi) = \frac{\prod_{j=1}^{M} \Gamma(b_j - \beta_j\xi) \prod_{j=1}^{N} \{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q} \{(1-b_j+\beta_j\xi)\}^{B_j} \prod_{j=N+1}^{P} (a_j - \alpha_j\xi)}$$
…(3)

and also the $\overline{\text{H}}$-function occurring in the paper was introduced by Inayat-Hussain [4] and studied by Bushman and Shrivastava [2]. The following series representation for the $\overline{\text{H}}$-function was obtained by Rathie [5].

$$\overline{H}_{P,Q}^{M,N}[z] = \overline{H}_{P,Q}^{M,N}\left[ z \left| \begin{matrix} (c_j,\gamma_j;C_j)_{1,N},(c_j,\gamma_j)_{N+1,P} \\ (d_j,\delta_j)_{1,M},(d_j,\delta_j;D_j)_{M+1,Q} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty}\sum_{h=1}^{M} \frac{\prod_{j=1, j\neq h}^{M} \Gamma(d_j - \delta_j\xi_{h,r}) \prod_{j=1}^{N} \{\Gamma(1-c_j+\gamma_j\xi_{h,r})\}^{C_j}}{\prod_{j=M+1}^{Q} \{\Gamma(1-d_j+\delta_j\xi_{h,r})\}^{D_j} \prod_{j=1}^{P} \Gamma(c_j-\gamma_j\xi_{h,r})} \frac{(-1)^r}{r!\,\delta_h} z^{\xi_{h,r}}$$

$$\xi_{h,r} = \frac{d_r + h}{\delta_r} \qquad ...(4)$$

The nature of contour L and series of various conditions on its parameters can be seen in the paper by Bushman and Shrivastava [2]. We shall also make use of the following behaviour of the $\overline{H}_{P,Q}^{M,N}[z]$ function for small value of f(z) as recorded by Saxena [6, p.112, eq.(2.3) and (2.4)]

$$\overline{H}_{P,Q}^{M,N}[z\} = 0(|z|^{\alpha}) \text{ for small z}$$

where $\gamma = \min_{1\leq j\leq M} \text{Re}(d_j/\delta_j)$ for (2)

and $\alpha = \min_{1\leq j\leq M} \text{Re}(b_j/\beta_j)$ for (4)

The following more general conditions given by

$$|\arg(z_1)| < \frac{1}{2}T_1\pi, \; T_1 > 0$$

and $$|\arg(z_2)| < \frac{1}{2}T_2\pi, \; T_2 > 0.$$

where

$$T_1 = \sum_{j=1}^{M_1}\beta_j + \sum_{j=1}^{N_1}|A_j\alpha_j| - \sum_{j=M_1+1}^{Q_1}|B_j\beta_j| - \sum_{j=N_1+1}^{P_1}\alpha_j > 0 \quad \text{and}$$

$$T_2 = \sum_{j=1}^{M_2}\delta_j + \sum_{j=1}^{N_2}|C_j\gamma_j| - \sum_{j=M_2+1}^{Q_2}|D_j\delta_j| - \sum_{j=N_2+1}^{P_2}\gamma_j > 0 \quad .$$

## II. MAIN INTEGRAL

$$\int_0^L \left(\sin\frac{\pi x}{L}\right)^{u-1} \sin\frac{\lambda_m \pi x}{L} S_{n_1}^{m_1}\left[y_1\left(\sin\frac{\pi x}{L}\right)^{b_1}\right] S_{n_2}^{m_2}\left[y_2\left(\sin\frac{\pi x}{L}\right)^{b_2}\right]$$

$$\overline{H}_{P_1,Q_1}^{M_1,N_1}\left[ z_1\left(\sin\frac{\pi x}{L}\right)^{h_1} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1} \\ (b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1} \end{matrix} \right. \right]$$

$$\overline{H}_{P_2,Q_2}^{M_2,N_2}\left[ z_2\left(\sin\frac{\pi x}{L}\right)^{h_2} \left| \begin{matrix} (c_j,\gamma_j;C_j)_{1,N_2},(c_j,\gamma_j)_{N_2+1,P_2} \\ (d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2} \end{matrix} \right. \right] dx$$

$$= k \sum_{k_1=0}^{[n_1/m_1]} \sum_{k_2=0}^{[n_2/m_2]} \sum_{r=0}^{\infty} \sum_{h_2=1}^{M_2} 2^{-(b_1k_1+b_2k_2+h_2\xi_{h,r})} \frac{(-n_1)_{m_1,k_1} A_{n_1,k_1}}{k_1!} y_1^{k_1}$$

$$\frac{(-n_2)_{m_2,k_2} A_{n_2,k_2}}{k_2!} y_2^{k_2} \sin\frac{\pi\lambda_m}{2}$$

$$\frac{\prod_{\substack{j=1\\ j\neq h}}^{M_2} \Gamma(d_j-\delta_j\xi_{h,r}) \prod_{j=1}^{N_2} \{\Gamma(1-c_j+\gamma_j\xi_{h,r})\}^{C_j}}{\prod_{j=M_2+1}^{Q_2} \{\Gamma(1-d_j+\delta_j\xi_{h,r})\}^{D_j} \prod_{j=N_2+1}^{P_2} \Gamma(c_j-\gamma_j\xi_{h,r})} \frac{(-1)^r z_2^{\xi_{h,r}}}{\delta_h \, r!}$$

$$\overline{H}_{P_1+1,Q_1+1}^{M_1,N_1+1}\left[ \frac{z_1}{2^h} \left| \begin{matrix} (-u-b_1k_1-b_2k_2-h_2\xi_{h,r}+1,h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1} \\ (b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1},(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}\mp\lambda_m,h_1;1) \end{matrix} \right. \right]$$

…(5)

Where (i) $k = L\,2^{-u+1}$

(ii) $h_1, h_2, n_1, n_2, k_1 \text{ and } k_2 > 0$ and

provided that conditions

(i) $\min \operatorname{Re}\left(\dfrac{b_j}{\beta_j}\right) > 0 \text{ and } \xi = \dfrac{b_h + r}{\beta_h}$

(ii) $\min \operatorname{Re}\left(\dfrac{d_j}{\delta_j}\right) > 0 \text{ and } \xi_{h,r} = \dfrac{d_h + r}{\delta_h}$

(iii) $\sum_{j=1}^{M_1} \beta_j + \sum_{j=1}^{N_1} |A_j\alpha_j| - \sum_{j=M_1+1}^{Q_1} |B_j\beta_j| - \sum_{j=N_1+1}^{P_1} \alpha_j \equiv T_1 > 0$

where $|\arg z_1| < \frac{1}{2} T_1 \pi$

(iv) $\sum_{j=1}^{M_2} \delta_j + \sum_{j=1}^{N_2} |C_j\gamma_j| - \sum_{j=M_2+1}^{Q_2} |D_j\delta_j| - \sum_{j=N_2+1}^{P_2} \gamma_j \equiv q_2 > 0$

where $|\arg z_2| < \frac{1}{2} T_2 \pi$

(v) $\operatorname{Re}\{u + b_1k_1 + b_2k_2 + h_2\xi_{h,r} + h_1\xi\} > 0$

Proof : To establish the above integral (5), we first express both the general class of polynomials and $\overline{H}_{p_2,Q_2}^{M_2,N_2}[z]$ occurring in its left hand side in their respective series forms with the help of equation (1) and (3) respectively and then interchange the order of integration and summation (which is permissible under the condition stated) and using (3) and with the help of x-integral given by Gradshteyn, I.S. and Ryzhik [3]. Then we substitute the above $\overline{\phi}(\xi)$ with the help of (4) and reinterpret the result thus in terms of $\overline{H}$-function, we arrive at the right hand side of desired results (5).

## III. MAIN PROBLEM

Problem of heat conduction in a rod with one end held at zero temperature and the other end exchanges heat freely with the surrounding medium at zero temperature. If the thermal coefficients are constants and there are no source of thermal energy, then temperature in a one-dimensional rod $0 \le x \le L$ satisfies the following heat equation

$$\frac{\partial\theta}{\partial t} = k\frac{\partial^2\theta}{\partial t^2},\ t \ge 0 \quad \text{…(6)}$$

In view of the problem, the solution of this partial differential equation satisfy the boundary conditions

$$\theta(0,t) = 0 \qquad \text{…(7)}$$

$$\frac{\partial\theta}{\partial t}(L,t) + h\,\theta(L,t) = 0 \quad \text{…(8)}$$

$\theta(x,t)$ is finite as $t \to \infty$ …(9)

The initial condition

$\theta(x,0) = f(x)$ …(10)

The solution of partial differential equation (6) can be written as [11, p.77,(4)]

$$\theta(x,t) = \sum_{m=1}^{\infty} B_m \sin\lambda_m \frac{\pi x}{L}\left\{ e^{-\left(\frac{\pi\lambda_m}{L}\right)^2 kt} \right\}$$

…(11)

at t = 0

$$\theta(x,0) = f(x) = \left(\sin\frac{\pi x}{L}\right)^{u-1} S_{n_1}^{m_1}\left[y_1\left(\sin\frac{\pi x}{L}\right)^{b_1}\right] S_{n_2}^{m_2}\left[y_2\left(\sin\frac{\pi x}{L}\right)^{b_2}\right]$$

$$\overline{H}_{P_1,Q_1}^{M_1,N_1}\left[ z_1\left(\sin\frac{\pi x}{L}\right)^{h_1} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1} \\ (b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1} \end{matrix} \right. \right]$$

$$\overline{H}_{P_2,Q_2}^{M_2,N_2}\left[ z_2\left(\sin\frac{\pi x}{L}\right)^{h_2} \left| \begin{matrix} (c_j,\gamma_j;C_j)_{1,N_2},(c_j,\gamma_j)_{N_2+1,P_2} \\ (d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2} \end{matrix} \right. \right]$$

…(12)

The solution of the problem to be obtained is

$$\theta(x,t) = k' \sum_{k_1=0}^{[n_1/m_1]} \sum_{k_2=0}^{[n_2/m_2]} \sum_{r=0}^{\infty} \sum_{h_2=1}^{m_2} 2^{-(b_1k_1+b_2k_2+b_2\xi_{h,r})}$$

$$\frac{(-n_1)_{m_1,k_1} A_{n_1,k_1}}{k_1!} y_1^{k_1} \frac{(-n_2)_{m_2,k_2} A_{n_2,k_2}}{k_2!} y_2^{k_2}$$

$$\frac{\prod_{\substack{j=1\\ j\neq h}}^{M_2}\Gamma(d_j-\delta_j\xi_{h,r})\prod_{j=1}^{N_2}\{\Gamma(1-c_j+\gamma_j\xi_{h,r})\}^{C_j}}{\prod_{j=M_2+1}^{Q_2}\{\Gamma(1-d_j+\delta_j\xi_{h,r})\}^{D_j}\prod_{j=N_2+1}^{P_2}\Gamma(c_j-\gamma_j\xi_{h,r})}\frac{(-1)^r z_2^{\xi_{h,r}}}{\delta_h\, r!}$$

$$\frac{\lambda_m \sin\left(\frac{\pi\lambda_m}{2}\right)}{2\pi\lambda_m-\sin 2\pi\lambda_m}\sin\left(\lambda_m\frac{\pi x}{L}\right)\exp\left\{-\left(\frac{\pi\lambda_m}{L}\right)^2 kt\right\}$$

$$\bar{H}^{M_1,N_1+1}_{P_1+1,Q_1+1}\left[\frac{z_1}{2^h}\left|\begin{matrix}(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}+1,h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\ (b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M+1,Q_1},(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}\mp\lambda_m,h_1;1)\end{matrix}\right.\right] \quad \ldots(13)$$

Where $k'=\frac{4\pi}{\sqrt{\pi d}}2^{-u+1}L$

valid under the condition of (5).

**Proof**: The solution of the problem stated is given as [11, p.77, (4)]

$$\theta(x,t)=\sum_{m=1}^{\infty} B_m \sin\lambda_m\frac{\pi x}{L}\left\{e^{-\left(\frac{\pi\lambda_m}{L}\right)^2 kt}\right\} \text{ where}$$

$\lambda_1, \lambda_2, \ldots$ are rots of transcendental equation.

$$\tan\pi\lambda_m=\frac{\pi\lambda_m}{kL}$$

If t = 0, then by virtue of (11) and (12), we have

$$\left(\sin\frac{\pi x}{L}\right)^{u-1} S_{n_1}^{m_1}\left[y_1\left(\sin\frac{\pi x}{L}\right)^{b_1}\right] S_{n_2}^{m_2}\left[y_2\left(\sin\frac{\pi x}{L}\right)^{b_2}\right]$$

$$\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[z_1\left(\sin\frac{\pi x}{L}\right)^{h_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\ (b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$\bar{H}^{M_2,N_2}_{P_2,Q_2}\left[z_2\left(\sin\frac{\pi x}{L}\right)^{h_2}\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N_2},(c_j,\gamma_j)_{N_2+1,P_2}\\ (d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]$$

$$=\sum_{m=1}^{\infty} B_m \sin\frac{\lambda_m \pi x}{L} \quad \ldots(13)$$

Multiplying both sides of (13) by $\sin\lambda_m\frac{\pi x}{L}$ and integrate with respect to x from 0 to L, we get

$$\int_0^L \left(\sin\frac{\pi x}{L}\right)^{u-1}\sin\lambda_m\frac{\pi x}{L} S_{n_1}^{m_1}\left[y_1\left(\sin\frac{\pi x}{L}\right)^{b_1}\right] S_{n_2}^{m_2}\left[y_2\left(\sin\frac{\pi x}{L}\right)^{b_2}\right]$$

$$\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[z_1\left(\sin\frac{\pi x}{L}\right)^{h_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\ (b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$\bar{H}^{M_2,N_2}_{P_2,Q_2}\left[z_2\left(\sin\frac{\pi x}{L}\right)^{h_2}\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N_2},(c_j,\gamma_j)_{N_2+1,P_2}\\ (d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]$$

$$=\sum_{m=1}^{\infty} B_m\int_0^L \sin\lambda_m\frac{\pi x}{L}\sin\lambda_m\frac{\pi x}{L}dx \quad \ldots(14)$$

and using (5) and orthogonality property [12, p.28] by Szego, we obtain

$$B_m=\frac{4\lambda_m\left(\frac{\pi}{d}\right)^{1/2}\sin\left(\frac{\pi\lambda_m}{2}\right)}{2\pi\lambda_m-\sin 2\pi\lambda_m}$$

$$k'\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\sum_{r=0}^{\infty}\sum_{h_2=1}^{m_2} 2^{-(b_1k_1+b_2k_2+b_2\xi_{h,r})}$$

$$\frac{(-n_1)_{m_1,k_1} A_{n_1,k_1}}{k_1!} y_1^{k_1} \frac{(-n_2)_{m_2,k_2} A_{n_2,k_2}}{k_2!} y_2^{k_2}\sin\frac{\pi\lambda_m}{2}$$

$$\frac{\prod_{\substack{j=1\\ j\neq h}}^{M_2}\Gamma(d_j-\delta_j\xi_{h,r})\prod_{j=1}^{N_2}\{\Gamma(1-c_j+\gamma_j\xi_{h,r})\}^{C_j}}{\prod_{j=M_2+1}^{Q_2}\{\Gamma(1-d_j+\delta_j\xi_{h,r})\}^{D_j}\prod_{j=N_2+1}^{P_2}\Gamma(c_j-\gamma_j\xi_{h,r})}\frac{(-1)^r z_2^{\xi_{h,r}}}{\delta_h\, r!}$$

$$\bar{H}^{M_1,N_1+1}_{P_1+1,Q_1+1}\left[\left(\frac{z_1}{2^h}\right)^{\xi}\left|\begin{matrix}(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}+1,h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\ (b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M+1,Q_1},(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}\mp\lambda_m,h_1;1)\end{matrix}\right.\right] \quad \ldots(15)$$

with the help of (14) and (11), we arrive at the right hand side of desired result.

## IV. EXPANSION FORMULA

$$\left(\sin\frac{\pi x}{L}\right)^{u-1} S_{n_1}^{m_1}\left[y_1\left(\sin\frac{\pi x}{L}\right)^{b_1}\right] S_{n_2}^{m_2}\left[y_2\left(\sin\frac{\pi x}{L}\right)^{b_2}\right]$$

$$\bar{H}^{M_1,N_1}_{P_1,Q_1}\left[z_1\left(\sin\frac{\pi x}{L}\right)^{h_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\ (b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$\bar{H}^{M_2,N_2}_{P_2,Q_2}\left[z_2\left(\sin\frac{\pi x}{L}\right)^{h_2}\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N_2},(c_j,\gamma_j)_{N_2+1,P_2}\\ (d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]$$

$$= k'\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\sum_{r=0}^{\infty}\sum_{h_2=1}^{m_2} 2^{-(b_1k_1+b_2k_2+b_2\xi_{h,r})}$$

$$\frac{(-n_1)_{m_1,k_1} A_{n_1,k_1}}{k_1!} y_1^{k_1} \frac{(-n_2)_{m_2,k_2} A_{n_2,k_2}}{k_2!} y_2^{k_2}$$

$$\frac{\prod_{\substack{j=1\\ j\neq h}}^{M_2}\Gamma(d_j-\delta_j\xi_{h,r})\prod_{j=1}^{N_2}\{\Gamma(1-c_j+\gamma_j\xi_{h,r})\}^{C_j}}{\prod_{j=M_2+1}^{Q_2}\{\Gamma(1-d_j+\delta_j\xi_{h,r})\}^{D_j}\prod_{j=N_2+1}^{P_2}\Gamma(c_j-\gamma_j\xi_{h,r})}\frac{(-1)^r z_2^{\xi_{h,r}}}{\delta_h\, r!}$$

$$\lambda_m\left(\sin\frac{\pi\lambda_m}{2}\right)\left(\sin\frac{\lambda_m\pi x}{L}\right)$$

$$\overline{H}^{M_1,N_1+1}_{P_1+1,Q_1+1}\left[\left(\frac{z_1}{2^h}\right)^{\xi}\left|\begin{matrix}(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}+1,h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M+1,Q_1},(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}\mp\lambda_m,h_1;1)\end{matrix}\right.\right] \quad \ldots(16)$$

where all the conditions of (5) are satisfied.

**Proof**: Using (12) and (15) in (11), we arrive at the expansion formula

$$k' = \frac{4\pi}{\sqrt{\pi d}} L\, 2^{-4+1}$$

**6. SPECIAL CASES OF (12)**

(i) If $\quad A_{n,k} = \binom{n+\alpha}{n}\dfrac{(\alpha+\beta+n+1)}{(\alpha+1)_k}$

then $\quad s'_n[x] \to P_n^{(\alpha,\beta)}(1-2x)$

where $P_n^{(\alpha,\beta)}(x)$ is Jacobi polynomial [10, p.68, eq. (4.3.2)] and also

$$m=1,\; A_{n,k} = \binom{n+\alpha}{n}\frac{1}{(\alpha+1)_k}$$

then $\quad S'_n[x] \to L^{(\gamma)}_{n_L}(x)$

where $L^{(\gamma)}_{n_L}(x)$ is Leguerre Polynomial [10, p.101, eq. (5.1.6)] and we get

$$f(x) = \left(\sin\frac{\pi x}{L}\right)^{u-1} P^{(\alpha,\beta)}_{n_1}\left(1-2\sin\frac{\pi x}{L}\right) L^{\gamma}_{n_2}\left(\sin\frac{\pi x}{L}\right)$$

$$\overline{H}^{M_1,N_1}_{P_1,Q_1}\left[z_1\left(\sin\frac{\pi x}{L}\right)^{h_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M_1+1,Q_1}\end{matrix}\right.\right]$$

$$\overline{H}^{M_2,N_2}_{P_2,Q_2}\left[z_2\left(\sin\frac{\pi x}{L}\right)^{h_2}\left|\begin{matrix}(c_j,\gamma_j;C_j)_{1,N_2},(d_j,\delta_j)_{N_2+1,P_2}\\(d_j,\delta_j)_{1,M_2},(d_j,\delta_j;D_j)_{M_2+1,Q_2}\end{matrix}\right.\right]$$

$$= k' \sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\sum_{r=0}^{\infty}\sum_{h_2=1}^{M_2} 2^{-(b_1k_1+b_2k_2+b_2\xi_{h,r})}\,.$$

$$\frac{(-n_1)_{m_1,k_1}}{k_1!}y_1^{k_1}\frac{(-n_2)_{m_2,k_2}}{k_2!}y_2^{k_2}$$

$$\frac{\lambda_m\sin\left(\dfrac{\pi\lambda_m}{2}\right)}{2\pi\lambda_m-\sin 2\pi\lambda_m}\sin\left(\frac{\lambda_m\pi x}{L}\right)\exp\left\{-\left(\frac{\pi\lambda_m}{L}\right)^2 kt\right\}$$

$$\frac{\prod\limits_{\substack{j=1\\j\neq h}}^{M_2}\Gamma(d_j-\delta_j\xi_{h,r})\prod\limits_{j=1}^{N_2}\{\Gamma(1-c_j+\gamma_j\xi_{h,r})\}^{C_j}}{\prod\limits_{j=M_2+1}^{Q_2}\{\Gamma(1-d_j+\delta_j\xi_{h,r})\}^{D_j}\prod\limits_{j=N_2+1}^{P_2}\Gamma(c_j-\gamma_j\xi_{h,r})}\frac{(-1)^r z_2^{\xi_{h,r}}}{\delta_h\, r!}$$

$$\binom{n_1+\alpha}{n_1}\left(\frac{(\alpha+\beta+n_1+1)_{k_1}}{(\alpha+1)_{k_1}}\right)\binom{n_2+\gamma}{n_2}\frac{1}{(\gamma+1)_{\gamma}}$$

$$\overline{H}^{M_1,N_1+1}_{P_1+1,Q_1+1}\left[\left(\frac{z_1}{2^h}\right)^{\xi}\left|\begin{matrix}(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}+1,h_1;1),(a_j,\alpha_j;A_j)_{1,N_1},(a_j,\alpha_j)_{N_1+1,P_1}\\(b_j,\beta_j)_{1,M_1},(b_j,\beta_j;B_j)_{M+1,Q_1},(-u-b_1k_1-b_2k_2-h_2\xi_{h,r}\mp\lambda_m,h_1;1)\end{matrix}\right.\right]$$

(ii) If we substitute

$m_1 = m_2 = 0, n_1 = n_2 = 0, A(0,k_1) = 1, A(0,k_2) = 1,$

$n_1 = n_2 \to 0, N_2 = P_2 = 0, M_2 = Q_2 = 1, h_2 \to 0$ and $A_j = B_j = 1, \overline{H}^{M_1,N_1}_{P_1,Q_1}$

then it reduces to Fox H-function and $\alpha_j = \beta_j = C\,(i = 1,\ldots,P;\; j = 1,\ldots,Q)$ it reduces to the well known Meijer's G-function by [9] in (12) then we get a known result given in [1].

On applying the same procedure as above in (16) , then we can establish the other known results.

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