

Characterization Techniques of Metamaterials

Mohit Anand

Department of Electrical Communication,
Indian Institute of Science,
Bangalore-560012, India

Abstract—This paper focuses on the numerous techniques that have been proposed over the years for metamaterial characterization. These techniques are categorized into analytical, field averaging and experimental methods, which provide various methods to determine the complex permittivity, complex permeability and refractive index of metamaterials.

Index Terms— Metamaterials (MTM), Dual negative materials (DNG), Bi-anisotropy.

I. INTRODUCTION

Material characterization is all about using special techniques and methods to investigate and show the character of material. The most general description of a linear medium requires the determination of 36 frequency dependent, complex constitutive parameters. Bi-anisotropic media are very general linear media, and the four material dyadics comprise together 36 parameters responsible for the magneto-electric behavior. Anisotropic, bi-isotropic, and isotropic media can be considered as subclasses of bi-anisotropic materials with 18, 4, and 2 degrees of freedom; respectively. The extraction of this large number of coefficients is complicated and not required for most of the practical structures.

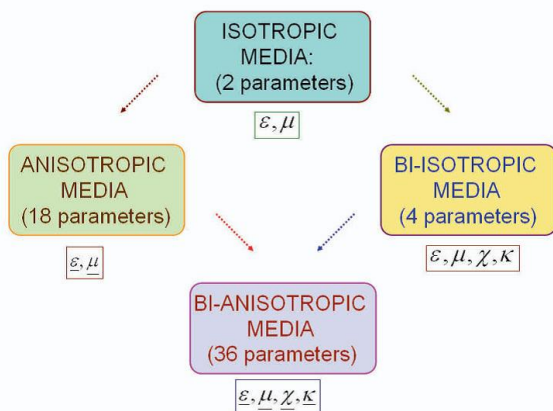


Fig. 1. Different types of materials.

Metamaterials provides properties which usually cannot be found in the nature; for example, simultaneous negative values of permittivity, permeability and refractive index. These properties have found many applications in antennas, optical communication, radar, microwaves and biomedical engineering.

The complete characterization of the effective medium is obtained by retrieving all the components of the permeability and permittivity tensors and accounting for both spatial and time dispersion. The capability of characterizing the anisotropy is especially useful in design and realization of metamaterials which requires a full control of all the components of the constitutive dyadic parameters. [1]

II. ANALYTICAL METHOD

Nowadays, an impressively large number of powerful characterization techniques are being used to solve analytical research problems; especially those related to the investigation of the properties of new materials for advanced applications. Analytical models give insight into the relationship between the physical properties and the geometrical characteristics of metamaterials with complicated structures. Metamaterials can be analyzed at two levels of abstraction:

Microstructures: described as a single unit cell of periodic lattice. The unit-cell analysis based on the following three steps:

- Extraction of effective constitutive parameters from scattering matrix with the method of parameter fitting of dispersive models.
- Analysis of the dispersion diagrams obtained by the solution of a periodic boundary eigen value problem.
- Higher order modes analysis based on the simulation of a multimode scattering matrix.

The above-mentioned steps, respectively provides:

- Effective electric permittivity and magnetic permeability in the frequency range of interest.
- Band structure of the periodic lattice with the frequency range and type (forward or backward) of the propagating mode.

(c) Frequency band, where the homogenized model of the metamaterial lattice is not valid due to the significant contribution of the higher order modes.

Macrostructures: Macrostructure approach allows for the observation of various interesting phenomena related to metamaterials and predicted from unit-cell level simulations. Macrostructure level simulations allow for prediction and visualization of the phenomena characteristic to metamaterials. The macrostructure can be simulated in two forms:

- (a) As a homogeneous effective structure or
- (b) As a rigorous detailed lattice

Effective medium theory was proposed by Lord Rayleigh in 1892. It was further studied and modified by Lorentz, Maxwell-Garnett and Bruggeman is used to characterize composite medium. This theory compares the average field propagating inside a composite medium with respect to field propagating inside a homogeneous medium to derive the electrical characteristics of the medium. The two main techniques to approximate the effective permittivity and conductivity of composite medium are:

A. Bruggeman's Model: Effective medium theories and approximations are developed from averaging the multiple values of the constituents that directly make up the composite material [2]. Bruggeman approximation assumes:

- Particles can still be considered spherical/ellipsoidal
 - assume particles experience 'average dielectric environment'
- Berremann model [3] approximates the effective permittivity as:

$$\frac{1}{n} \delta \alpha + \frac{(1-\delta)(\epsilon_m - \epsilon_{eff})}{\epsilon_m + (n-1)\epsilon_{eff}} = 0 \quad (1)$$

$$\alpha = \sum_{j=1}^n \left(\frac{(\epsilon - \epsilon_{eff})}{\epsilon_{eff} + L_j(\epsilon - \epsilon_{eff})} \right) \quad (2)$$

n → Euclidean spatial dimension

δ → Fraction of each component

ϵ_{eff} → Effective permittivity of medium

ϵ → Permittivity matrix

ϵ_m → Ellipsoidal/spherical inclusions of permittivity

L_j → Appropriate doublet/triplet of depolarization factors which is governed by the ratios between the axis of the ellipsoid.

B. Maxwell-Garnett Model:

This technique of effective medium approximation [4] consists of a matrix medium with permittivity ϵ_m and inclusions with permittivity ϵ_i .

$$\frac{\epsilon_{eff} - \epsilon_m}{\epsilon_{eff} + 2\epsilon_m} = \delta_i \left(\frac{\epsilon_i - \epsilon_m}{\epsilon_i - 2\epsilon_m} \right) \quad (3)$$

$$\epsilon_{eff} = \epsilon_m \frac{2(1-\delta_i)\epsilon_m + (1+2\delta_i)\epsilon_i}{(2+\delta_i)\epsilon_m + (1-\delta_i)\epsilon_i} \quad (4)$$

ϵ_{eff} → Effective permittivity of medium

ϵ_m → Matrix medium

ϵ_i → One of inclusions of permittivity

δ_i → Volume fraction of inclusions

C. Higher order mode analysis: The homogenized effective description and dispersion diagrams constitute valuable tools that allow one to predict properties of a metamaterial from the analysis of its single unit cell. However, the frequency ranges of stop-bands and pass-bands obtained by Eigen mode solver simulations do not always match the spectrum bands characterizing effective parameters retrieved from transmission and reflection coefficients[5] [6] [7].

D. Bloch mode analysis: Metamaterials occupy a special niche between homogeneous media and photonic crystals. For that reason, Bloch analysis and computation of band structures constitute important tools in the modeling of metamaterials As metamaterials do not rigorously satisfy the effective medium limit and are located conceptually between homogeneous materials and photonic crystals. [8]

E. Polarization and susceptibility Technique: This method was proposed in [9] and it uses the fact that the permittivity and permeability describe the interaction of a material with electric and magnetic fields. This behavior depends on the ability of the electromagnetic fields to polarize the particles in that material. Thus in this method, the susceptibility of the material is used to extract the effective material parameters (permittivity and permeability).

F. Circuit analysis Technique: This technique was proposed in [9] and it utilizes the circuit model of the material and uses those values to extract the material parameters from electromagnetic field equations. These field equations are mapped from the circuit telegrapher's equations via Ampere's law and the definition of potential. The permittivity and permeability are therefore relates directly to per unit length capacitance and inductance of a line [10].

III. FIELD AVERAGING METHODS

It is the most popular approach for the extraction of metamaterial constitutive parameters from transmission and reflection characteristics. Appropriate averages of local fields are obtained from analytic or full wave analysis within unit cell. This method provides accurate results when the dimensions of periodicity of unit cell are very small compare to the wavelength. Main drawback of this technique is that it does not take into account spatial dispersion and may fail in correctly predicting the scattering from metamaterial slab. Following three versions of field averaging techniques are available: Pendry's field averaging method, Smith's field averaging method and Acher's field averaging method. These approaches are particularly valuable for the study of metamaterials with gradient properties that can be modeled through electromagnetic simulation software. [11] [12] [13].

IV. EXPERIMENTAL CHARACTERIZATION METHOD

Metamaterials can be experimentally characterized by the following methods:

A. Nicolson-Ross-Weir Technique (NRW): This is the popular approach for the extraction of metamaterial constitutive parameters from transmission and reflection characteristics. These methods are commonly used in laboratories as an experimental way to find effective parameters of a material sample under test. This technique uses scattering parameters to derive the expressions of impedance and admittance of structures. These values can then be used to determine the reflection and transmission coefficients that can be used to determine the index of refraction and wave vector, which can be used for permittivity and permeability determination. [14] [15] [16]

B. Nicolson-Ross-Weir Variant Technique (NRW-Variant): This technique is similar to NRW technique but relations to extract the impedance and the index of refraction are different [17]. In this method, firstly the transmission coefficient, transmission coefficient and wave vector of incident wave are determined after that the complex expression for refractive index is determined from refractive index and impedance the values of effective permittivity and effective permeability are determined. Due to the explicit relation that results in unambiguous handedness of the resulting material, this technique is called NRW variant technique.

C. Resonator Method: This method provides high accuracy but it is narrowband in nature. Thus individual measurement setup should be required for retrieval of constitutive parameters.[18]

D. Free Space Method: This method requires very expensive setups.

E. Waveguide Toolkit Method: This method is based on transmission and reflection coefficient of the structure. It also requires expensive test setups.

F. Microstrip Topology: This method is a low cost and wide band method but it works only if effective permeability is positive hence not suitable for DNG materials.

G. Stripline Topology: This method is a low cost and wide band method and it works of all values effective permittivity and hence suitable for DNG materials.

V. CONCLUSION

In conclusion, we have seen that exact evaluation of the refractive index and constitutive parameters (permittivity, permeability, etc.) of metamaterials is in fact very difficult and the classical methods of retrieving the constitutive parameters have often led to non-physical results. By appropriately modeling the space dispersion and anisotropy, we can accurately predict the metamaterial constitutive parameters using suitable techniques.

ACKNOWLEDGMENT

This work has been supported by Space Applications Centre, Ahmedabad. The author wishes to thank Mr. K. J. Vinoy, Associate Professor, Department of Electrical communication, Indian Institute of Science, Bangalore, for useful discussions, valuable comments and guidance.

REFERENCES

- [1] J.B. Pendry, D. Schurig, and D.R. Smith, "Controlling electromagnetic fields", Science, vol. 312, pp. 1780-1782, June, 2006.
- [2] R. Landauer, "Electrical conductivity in inhomogeneous media," AIP Conf. Proc. 40, 2, 1978.
- [3] D.J. Bergman and K.J. Dunn, "Bulk effective dielectric constant of a composite with a periodic microgeometry", Physical Review B, Condens. Matter, vol. 45 no. 23, pp. 13262-13271, 1992.
- [4] J. Jamnik, J.R. Kalnin, E.A. Kotomin, J. Maier, "Generalised Maxwell-Garnett equation: application to electrical and chemical transport," Phys Chem Chem Phys. 2006 Mar 21;8(11):1310-4. Epub, Feb, 2006.
- [5] S. Foteinopoulou and C.M. Soukoulis, "Negative refraction and left-handed behavior in two-dimensional photonic crystals". Physical Review B, 67:235107(1-5), 22 Mar, 2003.

- [6] B. Gralak, S. Enoch, and G. Tayeb. „Anomalous refractive properties of photonic crystals”. Journal of the Optical Society of America A, 17(6):1012–1020, Jun, 2000.
- [7] D. Seetharamdoo, R. Sauleau, K. Mahdjoubi, and A.-C. Tarot.”Effective parameters of resonant negative refractive index metamaterials: interpretation and validity”. Journal of Applied Physics, 98:063505(1–4), 19 Sept, 2005.
- [8] A. Andryieuski, S. Ha, Andrey A. Sukhorukov, Y. S. Kivshar, A. V. Lavrinenko, “ Bloch-mode analysis for retrieving effective parameters of metamaterials,” Phys. Rev. B 86, 035127 , 2012
- [9] D. R. Smith, D.C. Vier, N. Kroll, and S. Schultz, “Direct calculation of permittivity and permeability for a left-handed metamaterial.” Applied Physics Letters 77(14), pp.2246-2248, October, 2000.
- [10] George V. Eleftheriades, Ashwin K. Iyer, and Peter C. Kremer, “Planar negative refractive index media using periodically 1-c loaded transmission lines.” IEEE Transactions on Microwave Theory and Techniques. 50(12): p.2702-2712, December, 2002.
- [11] D.R. Smith, J.B. Pendry, “Homogenization of metamaterials by field averaging”, Journal Optical Soc. Am., vol. 23, no. 3, pp. 391-403, 2006.
- [12] D. R. Smith,S. Schultz, P. Markos, and C.M. Soukoulis, ”Determination of effective permittivity and permeability of metamaterials from reflection and transmission coefficients.” Physical Review B. 65(195104):p.1-5, April, 2002.
- [13] D. R. Smith, D.C. Vier,N. Kroll, and S. Schultz, “Direct calculation of permittivity and permeability for a left-handed metamaterial.” Applied Physics Letters 77(14):p.2246-2248, October, 2000.
- [14] James Baker-Jarvis, Eric J. Vanzura, and William A. Kissick., “Improved technique for determining complex permittivity with the transmission/ reflection method.” IEEE Transactions on Microwave Theory and Techniques. 38(8):p.1096-1103, August 1990.
- [15] D.K. Ghodgaonkar,V.V. Varadan, and V.K. Varadan “Free space measurement of complex permittivity and complex permeability of magnetic materials at microwave frequencies,” IEEE Transactions on Instrumentation and Measurement,39(2): pp.387-394, April 1990.
- [16] Richard W. Ziolkowski, “Design fabrication, and testing of double negative metamaterials.” IEEE Transactions on Antenna and Propagation. 51(7):p.1516-1529, July, 2003.
- [17] D. R. Smith,S. Schultz, P. Markos, and C.M. Soukoulis, “Determination of effective permittivity and permeability of metamaterials from reflection and transmission coefficients .” Physical Review B. 65(195104):p.1-5, April, 2002.
- [18] Li, J.N, “Practical method for determining inductance and capacitance of metamaterial resonators,” Electronics Letters, vol. 48, pp.225-227, February, 2012.