International Journal of Technical Research and Applications e-ISSN: 2320-8163,
www.ijtra.com Volume 3, Issue 3 (May-June 2015), PP. 121-226

ON SEMI-ρ-CONTINUITY WHERE $\rho \in \{L, M, R, S\}$

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ABSTRACT: The authors Selvi.R, Thangavelu.P and Anitha.m introduced the concept of ρ-continuity between a topological space and a non empty set where $\rho \in$ {L, M, R, S} [4]. Navpreet singh Noorie and Rajni Bala[3] introduced the concept of $f^{\#}$ function to characterize the closed, open and continuous functions. In this paper, the concept of Semi-ρ-continuity is introduced and its properties are investigated and Semi-ρ-continuity is further characterized by using $f^{\#}$ functions.

KEYWORDS: Multifunction, saturated set, ρ-continuity, semi-open, semi-closed and continuity.

I. INTRODUCTION

By a multifunction F: X $\to$ Y, We mean a point to set correspondence from X into Y with $F(x) \neq \phi$ for all x $\in$ X. Any function f: X $\to$ Y induces a multifunction $f^{-1} \circ f$: X $\to \wp$(X). It also induces another multifunction $f \circ f^{-1}$: Y $\to$ (Y) provided f is surjective. The purpose is to introduced the notions of semi-L-Continuity, semi-M-Continuity, semi-R-Continuity and semi-S-Continuity of a function f: X $\to$ Y between a topological space and a non empty set. Here we discuss their links with semi-open and semi-closed sets. Also we establish pasting lemmas for semi-R-continuous and semi-S-continuous functions and obtain some characterizations for, semi-ρ-continuity. Navpreet singh Noorie and Rajni Bala [3] introduced the concept of $f^{\#}$ function to characterize the closed, open and continuous functions. The authors [6] characterized ρ-continuity by using $f^{\#}$- functions. In an analog way semi-ρ-continuity is characterized in this paper.

II. PRELIMINARIES

The following definitions and results that are due to the authors [4] and Navpreet singh Noorie and Rajni Bala [3] will be useful in sequel.

Definition: 2.1

Let f: (x, τ) $\to$ Y be a function. Then f is

(i) L-Continuous if $f^{-1}(f(A))$ is open in X for every open set A in X. [4]

(ii) M-Continuous if $f^{-1}(f(A))$ is closed in X for every closed set A in X. [4]

Definition: 2.2

Let f: X $\to$ (Y, σ) be a function. Then f is

(i) R-Continuous if $f(f^{-1}(B))$ is open in Y for every open set B in Y. [4]

(ii) S-Continuous if $f(f^{-1}(B))$ is closed in Y for every closed set B in Y. [4]

Definition 2.3:

Let f: X $\to$ Y be any map and E be any subset of X. then the following hold.

(i) $f^{\#}(E) = \{y \in Y: f^{-1}(y) \subseteq E\}$; (ii) $E^{\#} = f^{-1}(f^{\#}(E))$. [3]

Lemma 2.4:

Let E be a subset of X and let f: X $\to$ Y be a function. Then the following hold.

(i) $f^{\#}(E) = Y \backslash f(X \backslash E)$; (ii) $f(E) = Y \backslash f^{\#}(X \backslash E)$. [3]

Lemma 2.5:

Let E be a subset of X and let f: X $\to$ Y be a function. Then the following hold.

(i) $f^{-1}(f^{\#}(E)) = X \backslash f^{-1}(f(X \backslash E))$; (ii) $f^{-1}(f(E)) = X \backslash f^{-1}(f^{\#}(X \backslash E))$. [6]

Lemma 2.6:

Let E be a subset of X and let f: X $\to$ Y be a function. Then the following hold.

(i) $f^{\#}(f^{-1}(E)) = Y \backslash f(f^{-1}(Y \backslash E))$; (ii) $f(f^{-1}(E)) = Y \backslash f^{\#}(f^{-1}(Y \backslash E))$. [6]

Definition 2.7:

Let f: X $\to$ Y, A $\subseteq$ X and B $\subseteq$ Y. we say that A is f-saturated if $f^{-1}(f(A)) \subseteq A$ and B is f^{-1}-saturated if $f(f^{-1}(B)) \supseteq B$. Equivalently A is f-saturated if and only if $f^{-1}(f(A))=A$, and B is f^{-1}-saturated if and only if $f(f^{-1}(B)) = B$.

Definition 2.8:

Let A be a subset of a topological space(X, τ). Then A is called

(i) semi-open if $A \subseteq cl(int(A))$and semi-closed if $int(cl(A)) \subseteq A$;[1].

(ii) pre-open if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$;[2].

Definition: 2.9

Let f: (X, τ) $\to$ (Y, σ) be a function. Then f is semi-continuous if $f^{-1}(B)$ is open in X for every semi-open set B in Y, [1].

Definition: 2.10

Let f: (X, τ) $\to$ (Y, σ) be a function. Then f is semi-open (resp. semi-closed) if f(A) is semi-open(resp. semi-closed) in Y for every semi-open(resp. semi-closed) set A in X.

III. SEMI- ρ-CONTINUITY WHERE $\rho \in$ {L, M, R, S}

Definition: 3.1

Let f: (X, τ) $\to$ Y be a function. Then f is

(i) Semi-L-Continuous if $f^{-1}(f(A))$ is open in X for every semi -open set A in X.

(ii) Semi-M-Continuous if $f^{-1}(f(A))$ is closed in X for every semi-closed set A in X.

Definition: 3.2

Let f: X $\to$ (Y, σ) be a function. Then f is

(i) Semi-R-Continuous if $f(f^{-1}(B))$ is open in Y for every semi-open set B in Y.

(ii) Semi-S-Continuous if f (f $^{-1}$(B)) is closed in Y for every semi-closed set B in Y.

Example: 3.3

Let X = {a, b, c} and Y = {1, 2, 3}. Let τ ={ Φ , X ,{a}, {b}, {a, b}}.

Let f: $(X,\tau)\rightarrow Y$ defined by f(a)=2, f(b)=1, f(c)=3. Then f is Semi-L-Continuous and Semi-M-Continuous.

Example: 3.4

Let X = {a, b, c} and Y = {1, 2, 3}. Let σ ={ Φ,Y, {1},{2},{1,2}},

Let g : $X\rightarrow(Y,\sigma)$ defined by g(a)=2, g(b)=1, g(c)=3. Then g is Semi-R-Continuous and Semi-S-Continuous.

Definition: 3.5

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a function, Then f is

(i) semi -LR-Continuous, if it is both semi-L-Continuous and semi-R-Continuous.
(ii) semi-LS -Continuous, if it is both semi-L-Continuous and semi-S-Continuous.
(iii) semi-MR-Continuous, if it is both semi-M-Continuous and semi-R-Continuous.
(iv) semi-MS-Continuous, if it is both semi -M-Continuous and semi -S-Continuous.

Theorem: 3.6

(i) Every injective function f: $(X,\tau)\rightarrow(Y,\sigma)$ is semi-L-Continuous and semi-M-Continuous.
(ii) Every surjective function f:$(X,\tau)\rightarrow(Y,\sigma)$ is semi-R-Continuous and semi-S-Continuous.
(iii) Any constant function f: $(X,\tau)\rightarrow(Y,\sigma)$ is semi-R-Continuous and semi-S-Continuous.

Proof:

(i) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be injective function. Then semi-L-Continuity and semi-M-Continuity follow from the fact that f $^{-1}$(f (A)) =A. This proves (i).
(ii) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be surjective function. Since f is surjective, f (f $^{-1}$(B)) =B for every subset B of Y. Then f is both semi-R-Continuous and semi-S-Continuous. This proves (ii).
(iii) Suppose f(x) = y_o for every x in X. Then f (f $^{-1}$(B)) =Y if $y_o \in$ B and f (f $^{-1}$(B)) = Φ ,

if $y_o \in$ Y\B. This proves (iii).

Corollary: 3.7

If f: $(X,\tau)\rightarrow(Y,\sigma)$ be bijective function then f is semi-L-Continuous,

semi-M-Continuous, semi-R-Continuous and semi-S-Continuous.

Theorem: 3.8

Let f: $(X,\tau)\rightarrow(Y,\sigma)$.

(i) If f is L-Continuous (resp. M-Continuous) then it is semi-L-Continuous (resp. semi-M-Continuous).
(ii) If f is R-Continuous (resp. S-Continuous) then it is semi-R-Continuous (resp. semi-S-Continuous).

Proof:

(i) Let A $\subseteq$ X be semi-open (resp. semi-closed) in X. since every semi-open (resp. semi-closed) set is open (resp. closed) and since f is L-continuous (resp. M-continuous),
f $^{-1}$ (f(A)) is open (resp. closed) in X. Therefore f is semi -L-Continuous (resp. semi -M-Continuous).
(ii) Let B $\subseteq$ Y be semi-open (resp. semi-closed) in Y. since every semi-open (resp. semi-closed) set is open (resp. closed) and since f is R-continuous (resp. S-continuous),
f (f $^{-1}$(B)) is open (resp. closed) in Y. Therefore f is semi-R-Continuous (resp. semi-S-Continuous).

Theorem: 3.9

Let f: $(X,\tau)\rightarrow Y$ be semi-M-Continuous. Then int(cl(A)) is f-saturated whenever A is f-saturated and pre-open.

Proof:

Let A $\subseteq$ X be f-saturated. Since f is semi-M-Continuous, $\Rightarrow$ A is semi-closed set in X $\Rightarrow$ int(cl(A)) $\subseteq$ A. And since A is pre-open $\Rightarrow$ A $\subseteq$ int(cl(A)). Therefore int(cl(A)) = A. since A is f-saturated $\Rightarrow$ f $^{-1}$(f(A)) = A.

That implies int(cl(A))=f $^{-1}$(f(int(cl(A)))). Therefore Hence int(cl(A)) is f-saturated whenever A is f-saturated and pre-open.

Theorem: 3.10

Let f: $(X,\tau)\rightarrow Y$ be semi-L-Continuous. Then cl(int(A) is f-saturated whenever A is f-saturated and pre-closed.

Proof:

Let A $\subseteq$ X be f-saturated. Since f is semi-L-Continuous $\Rightarrow$ A is semi-open set in X $\Rightarrow$ A $\subseteq$ cl(int(A)). And since A is pre-closed $\Rightarrow$ cl(int(A)) $\subseteq$ A. Therefore cl(int(A))= A since A is f-saturated $\Rightarrow$ f^{-1}(f(A))= A. That implies cl(int(A))=f $^{-1}$(f(cl(int(A)))). Therefore Hence cl(int(A)) is f-saturated whenever A is f-saturated and pre-closed.

Theorem: 3.11

Let f: $X\rightarrow(Y,\sigma)$ be pre-S-Continuous. Then int(cl(B)) is f $^{-1}$ – saturated whenever B is f^{-1} –saturated and pre-open.

Proof:

Let B $\subseteq$ Y be f $^{-1}$ –saturated. Since f is pre-S-Continuous $\Rightarrow$ B is semi-closed set in Y $\Rightarrow$ int(cl(B)) $\subseteq$ B, and since B is pre-open $\Rightarrow$ B $\subseteq$ int(cl(B)), Therefore int(cl(B))=B, since B is f $^{-1}$ –saturated $\Rightarrow$ f(f $^{-1}$(B)) = B. which implies that f(f $^{-1}$(int(cl(B))))= int(cl(B)), Therefore hence int(cl(B)) is f $^{-1}$–saturated.

Theorem: 3.12

Let f: $X\rightarrow(Y,\sigma)$ be pre-R-Continuous Then cl(int(B)) is f $^{-1}$ – saturated whenever B is f $^{-1}$–saturated and pre-closed.

Proof:

Let B $\subseteq$ Y be f $^{-1}$-saturated. Since f is pre-R-Continuous $\Rightarrow$ B is semi-open set in Y $\Rightarrow$ B $\subseteq$ cl(int(B)), and since B is pre-closed $\Rightarrow$ cl(int(B)) $\subseteq$ B, Therefore cl(int(B))=B, since B is f $^{-1}$ –saturated $\Rightarrow$ f(f $^{-1}$(B)) = B. which implies that f(f^{-1}(cl(int(B))))= cl(int(B)), Therefore hence cl(int(B)) is f $^{-1}$–saturated.

IV. PROPERTIES

In this section we prove certain theorems related with semi-open and semi-closed functions.

Theorem: 4.1

(i) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be semi-open and semi-Continuous, Then f is semi-L-Continuous.
(ii) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be open and semi-Continuous, Then f is semi-R-Continuous.

Proof:

(i) Let $A \subseteq X$ be semi-open in X. Let f: $(X, \tau) \to (Y, \sigma)$ be semi-open and semi-Continuous.

since f is semi-open $\Rightarrow$ f(A) is semi-open in Y, and since f is semi-continuous, $\Rightarrow f^{-1}(f(A))$ is open in X. Therefore f is semi-L-Continuous, This proves (i).

(ii) Let $B \subseteq Y$ be semi-open in Y. Let f: $(X, \tau) \to (Y, \sigma)$ be open and semi-Continuous.

since f is semi-continuous $\Rightarrow f^{-1}(B)$ is open in X, and since f is open $\Rightarrow f(f^{-1}(B))$ is open in Y, Therefore f is semi-R-Continuous, This proves (ii).

Theorem: 4.2

(i) Let f: $(X, \tau) \to (Y, \sigma)$ be semi-closed and semi-Continuous, Then f is semi-M-Continuous.

(ii) Let f: $(X, \tau) \to (Y, \sigma)$ be closed and semi-Continuous, Then f is semi-S-Continuous.

Proof:

(i) Let $A \subseteq X$ be semi-closed in X. Let f: $(X, \tau) \to (Y, \sigma)$ be semi-closed and semi-continuous. since f is semi-closed $\Rightarrow$ f(A) is semi-closed in Y, and since f is semi-continuous, $\Rightarrow f^{-1}(f(A))$ is closed in X. Therefore f is semi-M-Continuous. This proves (i).

(ii) Let $B \subseteq Y$ be semi-closed in Y. Let f: $(X, \tau) \to (Y, \sigma)$ be closed and semi-Continuous. since f is semi-continuous $\Rightarrow f^{-1}(B)$ is closed in X, and since f is closed $\Rightarrow f(f^{-1}(B))$ is closed in Y. Therefore f is semi-S-Continuous, This proves (ii).

Theorem: 4.3

Let X be a topological space.

(i) If A is a semi-open subspace of X, the inclusion function j: A $\to$ X is semi-L-continuous and semi-R-continuous.

(ii) If A is a semi-closed subspace of X, the inclusion function j: A $\to$ X is semi-M-continuous and semi -S-continuous.

Proof:

(i) Suppose A is a semi-open subspace of X. Let j: A $\to$ X be an inclusion function. Let $U \subset X$ be semi-open in X then j $(j^{-1}(U))$ =j (U n A) =U n A Which is open in X. Hence j is semi-R-continuous. Now, let $U \subseteq A$ be semi-open in A. Then j^{-1} (j (U)) = $j^{-1}(U)$ =U which is open in A. Hence j is semi-L-continuous. This proves (i).

(ii) Suppose A is a semi-closed subspace of X. Let j: $A \to X$ be an inclusion function.

Let $U \subset X$ be semi-closed in X then $j(j^{-1}(U))$=j(U n A)=U n A, Which is closed in X. Hence j is semi-S-continuous. Now, let $U \subseteq A$ be semi-closed in A Then j^{-1} (j(U)) = $j^{-1}(U)$ =U which is closed in A. Hence j is semi-M-continuous. This proves (ii).

Theorem: 4.4

Let g: $Y \to Z$ and f: $X \to Y$ be any two functions. Then the following hold.

(i) If g: $Y \to Z$ is semi-L-continuous (resp. semi-M-continuous) and f: $X \to Y$ is semi-open (resp. semi-closed) and continuous, then $g \circ f$: $X \to Z$ is semi-L-continuous (resp. semi-M-continuous).

(ii) If g: $Y \to Z$ is open (resp. closed) and semi-continuous and f: $X \to Y$ is R-continuous (resp. S-continuous), then $g \circ f$ is semi-R-continuous (resp. semi-S-continuous).

Proof:

(i) Suppose g is semi-L-continuous (resp. semi-M-continuous) and f is semi-open (resp. semi-closed) and continuous. Let A be semi-open (resp. semi-closed) in X. Then $(g \circ f)^{-1}.(g \circ f)(A)=f^{-1}(g^{-1}(g(f(A))))$. Since f is semi-open (resp. semi-closed) $\Rightarrow$ f (A) is semi-open (resp. semi-closed) in Y. since g is semi-L-continuous (resp. semi-M-continuous),

$\Rightarrow g^{-1}(g(f(A)))$ is open (resp. closed) in Y, since f is continuous $\Rightarrow f^{-1}(g^{-1}(g(f(A))))$ is open (resp. closed) in X. Therefore, $g \circ f$ is semi-L-continuous (resp. semi-M-continuous). This proves (i).

(ii) Let f: X $\to$ Y be R-continuous (resp. S-continuous) and g: $Y \to Z$ be open (resp. closed) and semi-continuous. Let B be semi-open (resp. semi-closed) in Z. Then $(g \circ f)$ $(g \circ f)^{-1}(B)$ = $(g \circ f)$ $(f^{-1}(g^{-1}(B)))$ = $g(f(f^{-1}(g^{-1}(B))))$. since g is semi-continuous $\Rightarrow g^{-1}(B)$ is open (resp. closed) in Y. since f is R-continuous(resp. S-continuous)

$\Rightarrow f(f^{-1}(g^{-1}(B)))$ is open (resp. closed) in Y. since g is open (resp. closed) $\Rightarrow g(f(f^{-1}(g^{-1}(B))))$ is open (resp. closed)in Z. Therefore, $g \circ f$ is semi-R-continuous (resp. semi-S-continuous). This proves (ii).

Theorem: 4.5

If f: $X \to Y$ is semi-L-continuous and if A is an open subspace of X, then the restriction of f to A is semi-L-continuous.

Proof:

Let h = $f/_A$. Then h = $f \circ j$, where j is the inclusion map. j: $A \to X$. Since j is open and continuous and since f: $X \to Y$ is semi-L-continuous, using theorem (4.4 (i)), h is semi-L-continuous.

Theorem: 4.6

If f: $X \to Y$ is semi-M-continuous and if A is a closed subspace of X, then the restriction of f to A is semi-M-continuous.

Proof:

Let h = $f/_A$. Then h = $f \circ j$, where j is the inclusion map j: $A \to X$. Since j is closed and continuous and since f: $X \to Y$ is semi-M-continuous, using theorem (4.4 (i)), h is semi-M-continuous.

Theorem: 4.7

Let f: $X \to Y$ be semi-R-continuous. Let $f(x) \subseteq Z \subseteq Y$ and f(X) be open in Z. Let h: $X \to Z$ be obtained by from f by restricting the co-domain of f to Z. Then h is semi-R-continuous.

Proof:

Clearly h = $j \circ f$ where j: $f(x) \to Z$ is an inclusion map. Since f(X) is open in Z, the inclusion map j is both open and semi-continuous. Then by applying theorem (4.4(ii)), h is semi-R-continuous.

Theorem: 4.8

Let f: $X \to Y$ be semi-S-continuous. Let $f(x) \subseteq Z \subseteq Y$ and f(X) be closed in Z. Let h: $X \to Z$ be obtained by from f by restricting the co-domain of f to Z. Then h is semi-S-continuous.

Proof:

Clearly h = j $\circ$ f where j: f(x) $\rightarrow$ Z is an inclusion map. Since f(X) is closed in Z, the inclusion map j is both closed and semi-continuous. Then by applying theorem 4.4(ii), h is semi-S-continuous.

Now we establish the pasting lemmas for semi-R-continuous and semi-S-continuous functions.

Theorem: 4.9

Let X=A$\bigcup$B. Let f: A $\rightarrow$ (Y, σ) and g: B $\rightarrow$ (Y, σ) be semi-R-continuous (res. semi-S-continuous) f(x)=g(x) for every x $\in$ A$\bigcap$B, then f and g combined to give a semi-R-continuous (res. semi-S-continuous) function h: X $\rightarrow$ Y defined by h(x)=f(x) if x $\in$ A, and h(x)=g(x) if x $\in$ B.

Proof:

Let C be a semi-open (res. semi-closed) set in Y. Now $hh^{-1}(C)$ = h (f^{-1}(C) $\bigcup$ g^{-1}(C)) = h (f^{-1}(C)) $\bigcup$ h (g^{-1}(C)) = f (f^{-1}(C)) $\bigcup$ g (g^{-1}(C)). Since f is semi-R-continuous (res. semi-S-continuous), f (f^{-1}(C)) is open (resp. closed) in Y and Since g is semi-R-continuous (res. semi-S-continuous), g (g^{-1}(C)) is open (resp. closed) in Y. Therefore, hh^{-1}(C) is open (resp. closed) in Y. Hence, h is semi-R-continuous (res. semi-S-continuous).

V. CHARACTERIZATIONS

Theorem: 5.1

A function f: X $\rightarrow$ Y is semi-L-continuous if and only if f^{-1}($f^{\#}$(A)) is closed in X for every semi-closed subset A of X.

Proof:

Suppose f is semi-L-continuous. Let A be semi-closed in X. Then G = X\A is semi-open in X. since f is semi-L-continuous and since G is semi-open in X, $\Rightarrow$ f^{-1}(f(G)) is open in X. By applying lemma ((2.5)-(i)),

$\Rightarrow$ f^{-1}($f^{\#}$(A)) = X \ f^{-1}(f(X\A)) = X \ f^{-1}(f(G)). That implies f^{-1}($f^{\#}$(A)) is closed in X.

Conversely, we assume that f^{-1}($f^{\#}$(A)) is closed in X for every semi-closed subset A of X.

Let G be a semi -open in X. By our assumption, f^{-1} ($f^{\#}$(A)) is closed in X, where A = X\G.

By using lemma ((2.5)-(ii)) $\Rightarrow$ f^{-1}(f(G))= X \ f^{-1}($f^{\#}$(X\G))= X \ f^{-1}($f^{\#}$(A)).

That implies f^{-1} (f (G)) is open in X. Therefore, hence f is semi-L-continuous.

Theorem: 5.2

A function f: X $\rightarrow$ Y is semi-M-continuous if and only if f^{-1} ($f^{\#}$ (G)) is open in X for every semi-open subset G of X.

Proof:

Suppose f is semi-M-continuous. Let G be semi-open in X. Then A = X \G is semi-closed in X. since f is semi-M-continuous and since A is semi-closed in X $\Rightarrow$ f^{-1}(f(A)) is closed in X. By lemma ((2.5)-(i)),

$\Rightarrow$ f^{-1}($f^{\#}$(G))=X\ f^{-1}(f(X\G))= X\ f^{-1}(f(A)). That implies f^{-1}($f^{\#}$(G)) is open in X.

Conversely, we assume that f^{-1}($f^{\#}$(G)) is open in X for every semi-open subset G of X.

Let A be a semi-closed in X. By our assumption, f^{-1} ($f^{\#}$ (G)) is open in X, where G = X\A.

By using lemma ((2.5) - (ii)) $\Rightarrow$ f^{-1}(f(A)) = X \ f^{-1}($f^{\#}$(X\A)) = X \ f^{-1}($f^{\#}$(G)).

That implies f^{-1} (f (A)) is closed in X. Therefore, hence f is semi-M-continuous.

Theorem: 5.3

The function f: X $\rightarrow$ Y is semi-R-continuous if and only if $f^{\#}$ (f^{-1}(B)) is closed in Y for every semi-closed subset B of Y. Proof:

Suppose f is semi-R-continuous. Let B be semi-closed in Y. Then G=Y\B is semi-open in Y. since f is semi-R-continuous and since G is semi-open in Y,

$\Rightarrow$ f(f^{-1}(G)) is open in Y. Now by using lemma((2.6)(i)),

$\Rightarrow$ $f^{\#}$(f^{-1}(B))=Y\f(f^{-1}(Y\B))=Y\f(f^{-1}(G)). That implies $f^{\#}$ (f^{-1}(B)) is closed in Y.

Conversely, we assume that $f^{\#}$ (f^{-1}(B)) is closed in Y for every semi-closed subset B of Y.

Let G be semi-open in Y. Let B = Y\G. By our assumption, $f^{\#}$ (f^{-1}(B)) is closed in Y.

By lemma ((2.6)(ii)) $\Rightarrow$ f(f^{-1}(G)) = Y \ ($f^{\#}$(f^{-1}(Y\G))) = Y \ $f^{\#}$(f^{-1}(B)),

This proves that f(f^{-1}(G)) is open in Y. Therefore, hence f is semi-R-continuous.

Theorem: 5.4

The function f: X $\rightarrow$ Y is semi-S-continuous if and only if $f^{\#}$ (f^{-1}(G)) is open in Y for every semi-open subset G of Y.

Proof:

Suppose f is semi-S-continuous. Let G be semi-open in Y. Then B=Y\G is semi-closed in Y. Since f is semi-S-continuous and since B is semi-closed in Y $\Rightarrow$ f (f^{-1}(B)) is closed in Y. Now by using lemma ((2.6)(i))

$\Rightarrow$ $f^{\#}$(f^{-1}(G)) = Y \ f(f^{-1}(Y\G)) = Y\ f(f^{-1}(B)). That implies $f^{\#}$ (f^{-1}(G)) is open in Y. Conversely, we assume that $f^{\#}$ (f^{-1}(G)) is open in Y for every semi-open subset G of Y.

Let B be semi-closed in Y. Let G = Y\B. By our assumption, $f^{\#}$ (f^{-1}(G)) is open in Y. By lemma ((2.6) (ii)) $\Rightarrow$ f(f^{-1}(B)) = Y \ ($f^{\#}$(f^{-1}(Y\B))) = Y \ $f^{\#}$(f^{-1}(G)), This proves that f(f^{-1}(B)) is closed in Y. Therefore, hence f is semi-S-continuous.

Theorem: 5.5

Let f: (X, τ) $\rightarrow$ Y be a function. Then the following are equivalent.

(i) f is semi-L-continuous,
(ii) for every semi-closed subset A of X , f^{-1}($f^{\#}$(A) is closed in X,
(iii) for every x $\in$ X and for every semi-open set U in X with f(x) $\in$ f(U) there is an open set G in X with x ϵ G and f(G) $\subseteq$ f(U),
(iv) f^{-1}(f(cl(int(A)))) $\subseteq$ int(f^{-1}(f(A))) for every pre-closed subset A of X.
(v) cl(f^{-1}($f^{\#}$(A))) $\subseteq$ f^{-1}($f^{\#}$(int(cl(A)))) for every pre-open subset A of X.

Proof:

(i) $\Leftrightarrow$ (ii) : follows from theorem 5.1.

(i) $\Leftrightarrow$ (iii): Suppose f is semi-L-continuous.

Let U be semi-open set in X such that f(x) $\in$ f (U).

since f is semi-L-continuous, f^{-1}(f (U)) is open in X.

since x $\in$ f^{-1}(f(U)) there is an open set G in X, such that x $\in$ G $\subseteq$ f^{-1}(f(U))

$\Rightarrow$ f(G) $\subseteq$ f(f $^{-1}$(f(U))) $\subseteq$ f(U). This proves (iii).
conversely, suppose (iii) holds.
Let U be semi-open set in X and x $\in$ f $^{-1}$(f (U)). Then f(x) $\in$ f(U).
By using (iii), there is an open set G in X containing x such that f (G) $\subseteq$ f (U). Therefore x $\in$ G $\subseteq$ f $^{-1}$ (f (G)) $\subseteq$ f $^{-1}$(f(U)). That implies f $^{-1}$(f(U)) is open set in X,
This completes the proof for (i) $\Leftrightarrow$ (iii).
(i) $\Leftrightarrow$ (iv): Suppose f is semi-L-continuous.
Let A be a pre-closed subset of X. Then cl(int(A)) is semi-open set in X. By the semi-L-continuity of f $\Rightarrow$ f $^{-1}$ (f (cl(int(A)))) is open in X $\Rightarrow$ f $^{-1}$(f (cl(int(A)))) $\subseteq$ int(f $^{-1}$(f (cl(int(A))))).
since A is pre-closed in X $\Rightarrow$ f $^{-1}$(f(cl(int(A)))) $\subseteq$ f $^{-1}$(f(A)),
$\Rightarrow$ int(f $^{-1}$(f(cl(int(A))))) $\subseteq$ int(f $^{-1}$(f(A))),
It follows that f $^{-1}$ (f (cl(int(A)))) $\subseteq$ int(f $^{-1}$ (f (cl(int(A))))) $\subseteq$ int(f $^{-1}$(f(A))), This proves (iv).
Conversely, we assume that(iv) holds.
Let U be semi-open set in X $\Rightarrow$ f $^{-1}$(f(U)) $\subseteq$ f $^{-1}$(f(cl(int(U))))
since U is pre-closed by applying (iv) we get f $^{-1}$(f(cl(int(U)))) $\subseteq$ int(f $^{-1}$(f(U))),
Therefore f $^{-1}$(f(U)) $\subseteq$ int(f $^{-1}$(f(U))) and hence f $^{-1}$(f(U)) is open in X.
This proves that f is pre-L-continuous.
(ii) $\Leftrightarrow$ (v): Suppose (ii) holds. Let A be a semi-closed subset of X.
By using (ii) f $^{-1}$(f$^{\#}$(int(cl(A)))) is closed in X $\Rightarrow$ cl(f $^{-1}$(f$^{\#}$(int(cl(A))))) $\subseteq$ f $^{-1}$(f$^{\#}$(int(cl(A))))
since A is pre-open $\Rightarrow$ f $^{-1}$(f$^{\#}$(A)) $\subseteq$ f $^{-1}$(f$^{\#}$(int(cl(A)))) $\Rightarrow$ cl(f $^{-1}$(f$^{\#}$(A))) $\subseteq$ cl(f $^{-1}$(f$^{\#}$(int(cl(A)))))
it follows that cl(f $^{-1}$(f$^{\#}$(A))) $\subseteq$ f $^{-1}$(f$^{\#}$(int(cl(A)))). This proves (v).
Conversely, let us assume that (v) holds. Let A be a pre-open subset of X,
since A is semi-closed, by (v), we see that cl(f $^{-1}$(f$^{\#}$(A))) $\subseteq$ f $^{-1}$(f$^{\#}$(int(cl(A)))) $\subseteq$ f $^{-1}$(f$^{\#}$(A)), Therefore f $^{-1}$(f$^{\#}$(A)) is closed in X. This proves (ii).

Theorem: 5.6

Let f: (X, τ) $\rightarrow$ Y be a function. Then the following are equivalent.

(i) f is semi-M-continuous,
(ii) for every semi-open subset G of X, f $^{-1}$ (f$^{\#}$(G) is open in X.
(iii) cl(f $^{-1}$(f(A))) $\subseteq$ f $^{-1}$(f(int(cl(A)))) for every pre-open subset A of X.
(iv) f $^{-1}$(f$^{\#}$(cl(int(A)))) $\subseteq$ int(f $^{-1}$(f$^{\#}$(A))) for every pre-closed subset A of X.

Proof:
(i) $\Leftrightarrow$ (ii): follows from theorem 5.2.
(i) $\Leftrightarrow$ (iii) :Suppose f is semi-M-continuous. Let A be a pre-open set in X.
Then int(cl(A)) is semi-closed set in X.
Since f is semi-M-continuous, f $^{-1}$(f(int(cl(A)))) is closed in X,
$\Rightarrow$ cl(f $^{-1}$(f(int(cl(A))))) = f $^{-1}$(f(int(cl(A)))).
Since A is pre-open in X, we see that f $^{-1}$(f(A)) $\subseteq$ f $^{-1}$(f(int(cl(A)))),
it follows that, cl(f $^{-1}$(f(A))) $\subseteq$ cl(f $^{-1}$(f(int(cl(A))))) = f $^{-1}$(f(int(cl(A)))). This proves (iii).
Conversely, suppose (iii) holds.
Let A be semi-closed subset in X $\Rightarrow$ f $^{-1}$(f(int(cl(A)))) $\subseteq$ f $^{-1}$(f(A)). Since A is pre-open by applying (iii), cl(f $^{-1}$(f(A))) $\subseteq$ f $^{-1}$(f(int(cl(A)))),
it follows that cl(f $^{-1}$(f(A))) $\subseteq$ f $^{-1}$(f(int(cl(A)))) $\subseteq$ f $^{-1}$(f(A)) $\Rightarrow$ cl(f $^{-1}$(f(A))) $\subseteq$ f $^{-1}$(f(A)).
That implies f $^{-1}$(f(A)) is closed set in X. This completes the proof for (i) $\Leftrightarrow$ (iii).
(ii) $\Leftrightarrow$ (iv): Suppose (ii) holds.
Let A be a pre-closed subset of X. Then cl(int(A)) is semi-open in X. By (ii), f $^{-1}$(f$^{\#}$(cl(int(A)))) is open in X $\Rightarrow$ f $^{-1}$(f$^{\#}$(cl(int(A)))) $\subseteq$ int(f^{-1}(f$^{\#}$(cl(int(A)))))
since A is pre-closed in X $\Rightarrow$ f $^{-1}$(f$^{\#}$(cl(int(A)))) $\subseteq$ f $^{-1}$(f$^{\#}$(A))
$\Rightarrow$ int(f $^{-1}$(f$^{\#}$(cl(int(A))))) $\subseteq$ int(f $^{-1}$(f$^{\#}$(A))) we see that f $^{-1}$(f$^{\#}$(cl(int(A)))) $\subseteq$ int(f $^{-1}$(f$^{\#}$(A))). This proves (iv).
Suppose (iv) holds. Let G be semi-open in X $\Rightarrow$ f $^{-1}$(f$^{\#}$(G)) $\subseteq$ f^{-1}(f$^{\#}$(cl(int(G))))
since G is pre-closed in X, by using (iv) $\Rightarrow$ f $^{-1}$(f$^{\#}$(cl(int(G)))) $\subseteq$ int(f $^{-1}$(f$^{\#}$(G)))
we see that f $^{-1}$(f$^{\#}$(G)) $\subseteq$ int(f $^{-1}$(f$^{\#}$(G))).
Then it follows that f $^{-1}$(f$^{\#}$(G)) is open in X . This proves (ii).

Theorem: 5.7

Let f: X $\rightarrow$ (Y, σ) be a function and σ be a space with a base consisting of f $^{-1}$ saturated open sets. Then the following are equivalent.
(i) f is semi-R-continuous,
(ii) for every semi-closed subset B of X, f$^{\#}$(f $^{-1}$(B) is closed in Y,
(iii)for every x $\in$ X and for every semi-open set V in Y with x $\in$ f $^{-1}$(V) there is an open set G in Y with f(x) $\in$ G and f $^{-1}$(G) $\subseteq$ f $^{-1}$(V),
(iv) f(f $^{-1}$(cl(int(B)))) $\subseteq$ int(f(f $^{-1}$(B))) for every pre-closed subset B of Y.
(v) cl(f$^{\#}$(f $^{-1}$(B))) $\subseteq$ f$^{\#}$(f $^{-1}$(int(cl(B)))) for every pre-open subset B of Y.
proof:
(i) $\Leftrightarrow$ (ii): follows from theorem 5.3.
(i) $\Leftrightarrow$ (iii) :Suppose f is semi-R-continuous. Let V be a semi-open set in Y such that x $\in$ f $^{-1}$(V).
Since f is semi-R-continuous, f(f $^{-1}$(V)) is open in Y. f(x) $\in$ f(f $^{-1}$(V)) there is an open set G in Y such that f(x) $\in$ G $\subseteq$ f(f $^{-1}$(V)). That implies x $\in$ f $^{-1}$(G) $\subseteq$ f $^{-1}$(f(f $^{-1}$(V))) $\in$ f $^{-1}$(V). This proves (iii).
conversely, suppose (iii) holds. Let V be semi-open in Y and y $\in$ f(f $^{-1}$(G)).
Then y=f(x) for some x $\in$ f $^{-1}$(V).
By using (iii) there is an open set G in Y containing f(x) such that f $^{-1}$(G) $\subseteq$ f $^{-1}$(V).
We choose G to a f $^{-1}$–saturated in Y. Then G=f(f $^{-1}$(G)) $\subseteq$ f(f $^{-1}$(V)).
This proves that f(f $^{-1}$(V)) is open in Y. This proves that f is semi-R-continuous.
(i) $\Leftrightarrow$ (iv): Suppose f is semi-R-continuous. Let B be pre-closed subset in Y. $\Rightarrow$
cl(int(B)) is pre-closed set in Y. By the pre-R-continuity of f

$\Rightarrow$ f (f $^{-1}$(cl(int(B)))) is open in Y $\Rightarrow$ f (f $^{-1}$(cl(int(B)))) $\subseteq$ int(f (f $^{-1}$(cl(int(B))))) $\Rightarrow$ Since B is pre-closed in Y, We have f (f $^{-1}$(cl(int(B)))) $\subseteq$ f(f $^{-1}$(B)) $\Rightarrow$ int(f (f $^{-1}$(cl(int(B))))) $\subseteq$ int(f(f $^{-1}$(B)). Then it follows that f (f $^{-1}$(cl(int(B)))) $\subseteq$ int(f(f $^{-1}$(B))). This proves (iv).

Conversely, we assume that (iv) holds. Let B be semi-open set in Y $\Rightarrow$ f(f $^{-1}$(B)) $\subseteq$ f (f $^{-1}$(cl(int(B)))).

Since B is pre-closed by applying (iv), we get f(f $^{-1}$(cl(int(B)))) $\subseteq$ int(f(f $^{-1}$(B))).

Therefore f(f $^{-1}$(B)) $\subseteq$ int(f(f $^{-1}$(B))) and hence f(f $^{-1}$(B)) is open in Y.

This proves that f is semi-R-continuous.

(ii) $\Leftrightarrow$ (v): Suppose (ii) holds. Let B be a semi-closed subset of Y. By using (ii) f$^{\#}$(f $^{-1}$(int(cl(B)))) is closed in Y $\Rightarrow$ cl(f$^{\#}$(f $^{-1}$(int(cl(B))))) $\subseteq$ f$^{\#}$(f $^{-1}$(int(cl(B)))).

Since B is pre-open in Y $\Rightarrow$ f$^{\#}$(f $^{-1}$(B)) $\subseteq$ f$^{\#}$(f $^{-1}$(int(cl(B)))) $\Rightarrow$ cl(f$^{\#}$(f $^{-1}$(B))) $\subseteq$ cl(f$^{\#}$(f $^{-1}$(int(cl(B))))), it follows that cl(f$^{\#}$(f $^{-1}$(B))) $\subseteq$ f$^{\#}$(f $^{-1}$(int(cl(B)))), This proves (v).

Conversely, let us assume that (v) holds. Let B be a semi-closed subset of Y $\Rightarrow$ f$^{\#}$(f $^{-1}$(int(cl(B)))) $\subseteq$ f$^{\#}$(f $^{-1}$(B)).

Since B is pre-open in Y, by (v) $\Rightarrow$ cl(f$^{\#}$(f $^{-1}$(B))) $\subseteq$ f$^{\#}$(f $^{-1}$(int(cl(B)))),

$\Rightarrow$ cl(f$^{\#}$(f $^{-1}$(B))) $\subseteq$ f$^{\#}$(f $^{-1}$(B)), Therefore f$^{\#}$(f $^{-1}$(B)) is closed in Y, This proves (ii).

Theorem: 5.8

Let f: X $\rightarrow$ (Y, σ) be a function. Then the following are equivalent.

(i) f is semi-S-continuous,

(ii) for every semi-open subset V of Y, f$^{\#}$(f $^{-1}$(V)) is open in Y,

(iii) cl(f(f $^{-1}$(B))) $\subseteq$ f(f $^{-1}$(int(cl(B)))) for every pre-open subset B of Y.

(iv) f$^{\#}$(f $^{-1}$(cl(int(B)))) $\subseteq$ int(f$^{\#}$(f $^{-1}$(B))) for every pre-closed subset B of Y.

Proof:

(i) $\Leftrightarrow$ (ii): follows from theorem 5.4.

(i) $\Leftrightarrow$ (iii) :Suppose f is semi-S-continuous. Let B be a pre-open set in Y, therefore int(cl(B)) is semi-closed in Y, Since f is pre-S-continuous $\Rightarrow$ f(f $^{-1}$(int(cl(B)))) is closed in Y, $\Rightarrow$ cl(f(f $^{-1}$(int(cl(B))))) $\subseteq$ f(f $^{-1}$(int(cl(B)))).

Since B is pre-open in Y $\Rightarrow$ f(f $^{-1}$(B)) $\subseteq$ f(f $^{-1}$(int(cl(B)))) $\Rightarrow$ cl(f(f $^{-1}$(B))) $\subseteq$ cl(f(f $^{-1}$(int(cl(B))))) it follows that, cl(f(f $^{-1}$(B))) $\subseteq$ f(f $^{-1}$(int(cl(B)))), This proves (iii).

conversely, suppose (iii) holds. Let B be semi-closed subset in Y, $\Rightarrow$ f(f $^{-1}$(int(cl(B)))) $\subseteq$ f(f $^{-1}$(B)).

Since B is pre-open by applying(iii),cl(f(f $^{-1}$(B))) $\subseteq$ f(f $^{-1}$(int(cl(B))))

$\Rightarrow$ cl(f(f $^{-1}$(B))) $\subseteq$ f(f $^{-1}$(B)),

That implies f(f $^{-1}$(B)) is closed set in Y. This completes the proof for (i) $\Leftrightarrow$ (iii). (ii) $\Leftrightarrow$ (iv):

Suppose (ii) holds. Let B be a pre-closed subset of Y. Then cl(int(B)) is semi-open in Y. By (ii), f$^{\#}$(f $^{-1}$(cl(int(B)))) is open in Y $\Rightarrow$ f$^{\#}$(f $^{-1}$(cl(int(B)))) $\subseteq$ int(f$^{\#}$(f $^{-1}$(cl(int(B))))).

Since B is pre-closed in Y $\Rightarrow$ f$^{\#}$(f $^{-1}$(cl(int(B)))) $\subseteq$ f$^{\#}$(f $^{-1}$(B)) $\Rightarrow$ int(f$^{\#}$(f $^{-1}$(cl(int(B))))) $\subseteq$ int(f$^{\#}$(f $^{-1}$(B))).

we see that f$^{\#}$(f $^{-1}$(cl(int(B)))) $\subseteq$ int(f$^{\#}$(f $^{-1}$(B))). This proves (iv).

Suppose (iv) holds. Let V be semi-open in Y $\Rightarrow$ f$^{\#}$(f $^{-1}$(V)) $\subseteq$ f$^{\#}$(f $^{-1}$(cl(int(V)))).

Since V is pre-closed in Y, by using (iv), $\Rightarrow$ f$^{\#}$(f $^{-1}$(cl(int(V)))) $\subseteq$ int(f$^{\#}$(f $^{-1}$(V))) $\Rightarrow$ f$^{\#}$(f $^{-1}$(V)) $\subseteq$ int(f$^{\#}$(f $^{-1}$(V))),

Then it follows that f$^{\#}$(f $^{-1}$(V)) is open in Y. This proves (ii).

VI. CONCLUTION

In this paper the notions of semi-L-Continuity, semi-M-Continuity, semi-R-Continuity and semi-S-Continuity of a function f: X $\rightarrow$ Y between a topological space and a non-empty set are introduced. The purpose of this paper is to introduce, semi-ρ-continuity. Here we discuss their links with semi-open, semi-closed sets. Also we establish pasting lemmas for semi-R-continuous and semi-s-continuous functions and obtain some characterizations for, semi-ρ-continuity. We have put forward some examples to illustrate our notions

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