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DIFFICULTY OF PREDICTING EARTHQUAKES IN MOSUL DAM

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Abstract— **In this paper we investigate the chaotic behavior of time series of Earthquakes regressive in Mosul Dam. Measurements and indicators of statistics that are applied, using a binary technical test, the 0-1 test for chaos, as well as identify some of statistical and chaotically characteristics features for the real observations. The detail analysis shows through this work that the time series Earthquake regressive in Mosul dam is not independent and behave non-periodic chaotic and irregular. its behavior cannot be predicted in the long time.**

Index terms- **Chaotic, Time Series, Earthquake, Statistics, 0 --1 test,**

I. INTRODUCTION

Chaos is present in mechanical many phenomenon such as oscillators, electrical circuits, lasers, chemical reactions, nerve cells, heated fluids and weather system [14]. In the last two decades many studies have done on chaos theory in almost all the branches of geophysics(e.g. meteorological data, etc) [8], In this paper we investigate the presence of chaotic behavior in the time series of the magnitude of the earthquakes in Mosul dam, one of the seismo regions of Iraq. The predictability of a dynamic system is depended on its state (i.e. stationary,non stationary or chaotic state). Various methods of prediction are proposed based on linear and nonlinear theories. But yet the existence of chaos is not investigated in seismological data. The study of statistical and chaotic analysis for any dynamic phenomenon is of great importance for researchers to learn the characteristics of those phenomena and knowledge of their behavior The diagnosis of chaotic behavior of any time series helps to knows, that it is very difficult to build a function of the probability, difficult to forecast, predict its behavior in the long term and there is no usefulness in trying to find and estimate the order [15] . The technical traditional distinction between the dynamics of regular and chaotic is to calculate the laypanov exponent (λ). If the laypanov exponent $\lambda > 0$,it is indicator to chaos, where the convergent paths diverge exponential, if $\lambda < 0$ the converging paths remain close to each other, this method is often used for dynamic systems known equations [4]. If the equations are not known, the test of the experimental data in this way is an indirect, as the laypanov exponent estimated by using the theory of embedding dimension [16]. we or approximate linear for the factor of evolution [1]. To overcome this issue use, the binary test for chaos [9] , [10]. As well as identifing some features and characteristics of statistical and chaotic observation realism using the techniques and computer simulations by time series curve, state-space diagram, periodicity is identified by the spectral analysis. Check independence based on the correlation integral and free statistics by using the statistical indicator BDS, and get useful results in the diagnosis of behavior .

II. THE BINARY TEST FOR CHAOS

Recently a new test for determining chaos was introduced by Gottwald and Melbourne [10] . In contrast to usual method of computing maximal lyapunov exponent . Their method is applied directly to the time series data and does not require phase-space reconstruction. Moreover, the dimension and origin of the dynamical system and the form of the underlying equations are irrelevant. The input is the time series data and the output is 0 or 1, depending on whether the dynamics is non-chaotic or chaotic.

Description of the 0-1 test for chaos [11]

Consider scalar observable $\phi(j)$, $\phi(j)$, $j = 1,2,......T$ a discrete set of measurement data . perform the following sequence steps :

I. for $c \in (o,\pi)$, compute the translation variables

$$p_c(t) = \sum_{j=1}^{t} \phi(j) \cos jc \,, \quad q_c(t) = \sum_{j=1}^{t} \phi(j) \sin jc \qquad ...(1)$$

for $t = 1,2,.....T$.

Claim that

A. $p_c(t)$ and $q_c(t)$ is bounded if the underlying dynamics is non chaotic (e.g periodic or quasi periodic).

B. $p_c(t)$ and $q_c(t)$ behaves asymptotically like Brownian motion if the underlying dynamics is chaotic .

II. the mean square displacement of the translation variables $p_c(t)$, $q_c(t)$ defined in equation (1) , for several values of c . The mean square displacement is defined as

$$M_c(t)=\lim_{T\to\infty}\frac{1}{T}\sum_{j=1}^{T}\left[p_c(j+t)-p_c(j)\right]^2+\left[q_c(j+t)-q_c(j)\right]^2$$

and smoothing mean square displacement is

$$M_c(t)_{new}=M_c(t)_{old}-\frac{1-\cos(tc)}{1-\sin(tc)}\left(\lim_{T\to\infty}\frac{1}{T}\sum_{j=1}^{T}\phi(j)\right)^2 \qquad \ldots(2)$$

Such that $t<<T$, $t\le t_{cut}$, $t_{cut}<T$, where $t_{cut}<T$ The limit is assured by calculating In practice we find that $t_{cut}=T/10$ yields good results.

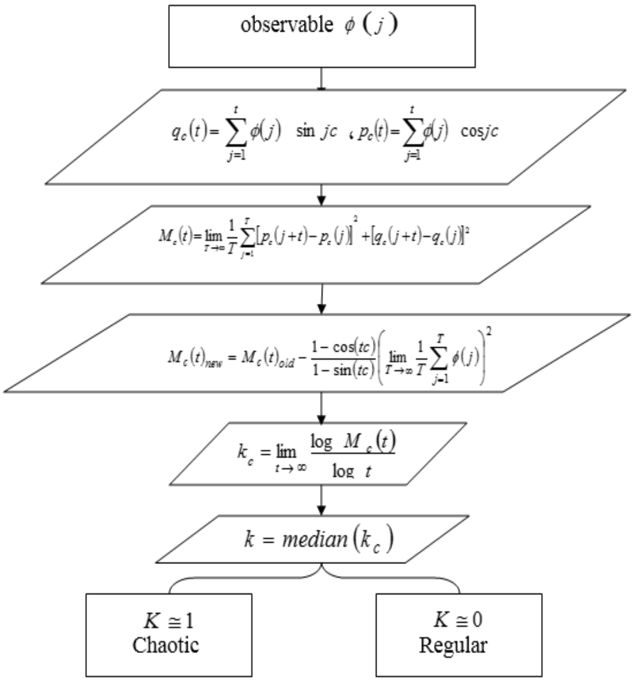
III- $M_c(t)$ grows linearly in time the underlying dynamic is chaotic , but it is not chaotic when $M_c(t)$ is bounded .

IV- The asymptotic growth rate k_c can be numerically determined by linear regression of $\log\ M_c(t)$ versus $\log(t)$, and is given by

$$k_c=\lim_{t\to\infty}\frac{\log M_c(t)}{\log t}$$

k = median (k_c) , if k ≈ 0 the system is non chaotic and if k ≈ 1 the system is chaotic .

Flow chart



III. - CORRELATION DIMENSION

The correlation dimension suggested by physicists Grassberger and Procaccia [12] , for a measure of chaos and now uses this measure on a large scale, for ease of handling its own and with the experimental data. Correlation dimension δ is one of the easiest metrics used to calculate the fractal dimension and deal with a observation given time series δ(t) ,and find sequence of vectors.

$$X(t)=(\delta(t),\delta(t+\tau),\ldots.\delta(t+(m-1))\tau) \qquad \ldots(3)$$

Where m is the dimension of phase space (in which the attractive are embedding and called the embedding dimension) . That all the analysis in time series depends on the Embedding Dimension, as well as rely on the Correlation Integral is defined as the following :

$$C(\varepsilon,m)=\frac{2}{N(N-1)}\sum_{j=1}^{N}\sum_{i=j+1}^{N}\theta(\varepsilon-\|X_i-X_j\|) \qquad \ldots.(4)$$

Where θ is Heavisido Function

$$\theta(x)=\begin{bmatrix}1 & \text{if} & x\ge 0\\ 0 & & \text{other wise}\end{bmatrix} \qquad \ldots.(5)$$

C(ε,m) is measures the probability of pairs converged in the time series for any small value (ε) and C(ε) is monotone increasing function from zero to one , C(ε) grow exponent by M_c i.e

$$C(\varepsilon)\approx\varepsilon^{Mc} \qquad \ldots.(6)$$

Obtain Grassberger and Procaccia [12], when N is very large, ε is very small. Logarithm of the correlation Integral as function of the logarithm of the length of the linear correlation has a tendency Mc, where Mc is the correlation dimension

$$M_c = \lim_{\varepsilon\to 0}\lim_{N\to\infty}\frac{\ln C_N(\varepsilon,m)}{\ln \varepsilon} \quad \text{....(7)}$$

The correlation dimension is lower when the value of the embedding dimension (m) when values the correlation dimension are converge[12].

Note 1: If the correlation dimension

1 - equals non-integer number, the situation is chaotic (Chaos)
2 - equals infinity number, in the case behavior is (Noise)
3 - equals integer number, the situation be linear (Linear) [4], [12] .

IV. Indicator of the Statistical BDS

BDS statistical indicator known in the mid-1988, and announced its programming by (C), language 1996 by scientists, Brock, Dechert, Scheinksmanh as the index is to examine the independence of time series

data to determine that the independent and have the same probability distribution or not. Not only the cursor on the BDS statistical non-linear systems and chaos theory, but used in the diagnosis of residual diagnostic systems in random [7] .

A. A Test for Independence [6]

Let $\{u_t\}$ random variables with Strictly Stationary , F is distribution function. Let

$$u_t^m = (u_t, u_{t+1}, \ldots\ldots, u_{t+m-1})$$

Its distribution function Fm, where $\{u_t\}$ is independent and

$$F_m(x_1,\ldots\ldots,x_m) = \prod_{k=1}^{m} F(x_k)$$

Suppose that

$$G_i^j = \delta - \{u_i, u_{i+1}, \ldots, u_j\} \qquad 1 \le i < j \le \infty$$

Stochastic process {ut} be regular, if the absolute

$$B_k = \sup_{n\ge 1}\left\{E\left[\sup\left\{\left|p(A/G_1^n) - p(A)\right| A \in G_{n+k}^{\infty}\right\}\right]\right\} \quad \text{....(8)}$$

The equation (8) close to zero. Such that $x \in R_m$, we use a more natural ((Max norm))

$$\|x\| = \max_{1\le k\le m}\{|X_k|\}$$

Distinctive function of the group A is χ_A , in particular case A=[0,ε). the Distinctive function called to χ_ε if the $\varphi: R^m \to R^l$ differentiation and $(D\phi)_\chi \in R^m$ is vector of partial derivatives for φ when $x \in R^m$,the Distinctive function in distinctive v

$$(D\varphi)_x v = \lim_{\varepsilon\to 0}\frac{\varphi(x+\varepsilon v) - \varphi(x)}{\varepsilon} \quad \text{....(9)}$$

B. The Correlation Integral

The correlation integral is more important in the statistical indicator BDS statistical and from applications of the correlation integral and other measures in nonlinear science test the hypothesis of Identical Independent Distributed (iid) [6]. This means that the null hypothesis be {Ho: Xt is iid} .

As the time series Xt, Ho is the null hypothesis [12]. We have correlation integral identified as a method to measure the fractal dimension of Deterministic data and correlation integral for Embedding dimension (m) given by

$$C_{m,n}(\varepsilon) = \frac{1}{\binom{n}{2}}\sum\sum_{1\le s\le t\le n}\chi_\varepsilon\left(\|u_s^m - u_t^m\|\right) \quad \text{....(10)}$$

And also

$$C_m(\varepsilon) = \lim_{n\to\infty} C_{m,n}(\varepsilon) \quad \text{....(11)}$$

If the data is generated from the stochastic process with Strictly stationary, strength never be regular, in this case the limit of the equation (6) be

$$C_m(\varepsilon) = \int\int \chi_\varepsilon\left(\|u - v\| dF_m(u).dF_m(v)\right) \quad \text{....(12)}$$

And when they are independent, and since the

$$\chi_\varepsilon\left(\|u - v\|\right) = \prod_{i=1}^{m}\chi_\varepsilon\left(|u_i - v_i|\right) \quad \text{....(13)}$$

Equation (12) are as

$$C_m(\varepsilon) = (C_1(\varepsilon))^m \quad \text{....(14)}$$

C. Asymptotic Distribution of Correlation Integral

D. Theorem : [5]

Let $\{u_t\}$ random variables of Identical Independent Distributed (iid), if the $k(\varepsilon) > C(\varepsilon)^2$, where

$$k(\varepsilon) = \int\left(\int \chi_\varepsilon\left(|u - v|\right) dF(u)\right)^2 dF(v) = \int\left[F(u+\varepsilon) - F(u-\varepsilon)\right]^2 dF(u) \quad \text{....(15)}$$

Then

$$\sqrt{n}\,\frac{C_{m,n}(\varepsilon) - (C_1(\varepsilon))^m}{\sigma_m(\varepsilon)} \quad \text{....(16)}$$

Close in distribution to N (0,1), Where

$$\frac{1}{4}\sigma_m^2 = k^m - C^{2m} + 2\sum_{i=1}^{m-1}\left[k^{m-i}C^{2i} - C^{2m}\right] \quad \text{....(17)}$$

Proof : - fond in [5].

4-3-2 Theorem : [5]

Let $\{u_t\}$ is (iid), if $K(\varepsilon) > C(\varepsilon)^2$ then $m \geq 2$

$$W_{m,n}(\varepsilon) = \sqrt{n}\frac{C_{m,n}(\varepsilon) - (C_1(\varepsilon))^m}{\sigma_m(\varepsilon)} \qquad ....(18)$$

Close in distribution to N (0,1), Where asymptotic distribution given by

$$\frac{1}{4}V_m^2 = m(m-2)C^{2m-2}(K - C^2) + K^m - C^{2m} + 2\sum_{j=1}^{m-1}\left[C^{2j}(K^{m-j} - C^{2m-2j} - mC^{2m-2}(K - C^2)\right] \qquad ....(19)$$

Proof : - fond in [5].

Note 2 : $W_{m,n}(\varepsilon)$ In the theorem (4.3.2) is the free distribution of statistics advantages are that no distributional assumptions are used as a laboratory for statistical independent random variables that have the same probability distribution [5]
.

V. Observed of Earthquakes in Mosul Dam

Mosul Dam is the largest dam in Iraq. It is located on the Tigris River in the western governorate of Nineveh, upstream of the city of Mosul. The hydroelectric dam holds, at full capacity, about 11.1 cubic kilometers (2.7 cu mi) of water and provides electricity to the 2.7 million residents of Mosul. The dam's main 750 MW power station contains four 187.5-MW Francis turbine-generators. A pumped-storage hydroelectricity power plant with a capacity of 240 MW and a run-of-the-river dam downstream with a 62 MW capacity also belong to the Mosul Dam scheme. It is ranked as the fourth largest dam in the Middle East, measured by reserve capacity, capturing snowmelt from Turkey, some 70 miles (110 km) north. Built on a karst foundation, concerns over the dam's instability have led to major remediation and rehabilitation efforts.

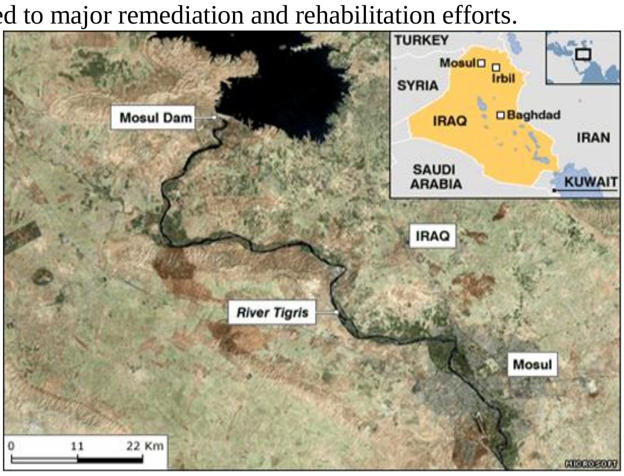


Figure (1): Time series plots to Observed of earthquakes in Mosul Dam in Two years (1986-1987).

The earthen embankment dam is located on top of gypsum, a soft mineral which dissolves in contact with water. Continuous maintenance is required to plug, or "grout" new leaks with a liquefied slurry of cement and other additives. More than 50,000 tonnes (49,000 long tons; 55,000 short tons) of material have been injected into the dam since leaks began forming shortly after the reservoir was filled in 1986, Mosul Dam is the most dangerous dam in the world." The report further outlined a worst case scenario, in which a sudden collapse of the dam would flood Mosul under 65 feet (20 m) of water and Baghdad, a city of 7 million, to 15 feet (5 m), with an estimated death toll of 500,000 [3]. A report on 30 October 2007 by the US Special Inspector General for Iraq Reconstruction (SIGIR) said that the dam's foundations could give way at any moment. This Dam cause earthquakes regressive, which tends to be non-significant, The Observed of earthquakes in Mosul Dam in Two years (1986-1987).

Figure (2): time series plots to Observed of earthquakes in Mosul Dam in Two years (1986-1987).

VI. Statistical Properties Observed of Earthquakes in Mosul Dam

In this section the most important features and statistical properties of the observed earthquakes in Mosul Dam, which is one of the indicators and benchmarks for the study and diagnosis of the chaotic behavior . The graphs and numerical results of computer simulation has been building and designing a language MATLAB.

A. Time Series Plots

Graphic of time series in Figure (3) refers to the behavior of irregular and complex observed of earthquakes.

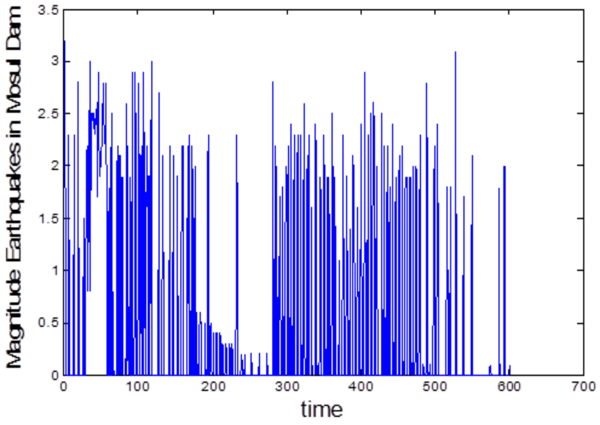


Figure (3): time series plots to observed of earthquakes in Mosul Dam in Two years (1986-1987).

B. State-Space Diagram

The state-space diagram, which is $x(t)$ compared $x(t+h)$ to each $h \geq 1$ of the indicators of good conduct time series, when the behavior is chaotic, the state-space in the form of a large group of curves (Contours) closed, which does not show the rate it is clear [2] [13].

The diagram two-dimensional state-space and the three dimensions of the observed of earthquakes in Mosul Dam in the figure (4) indicates that its behavior is irregular and non-periodic and has a complex interdependence where a large group of curves, as there is a difference in most of those curves.

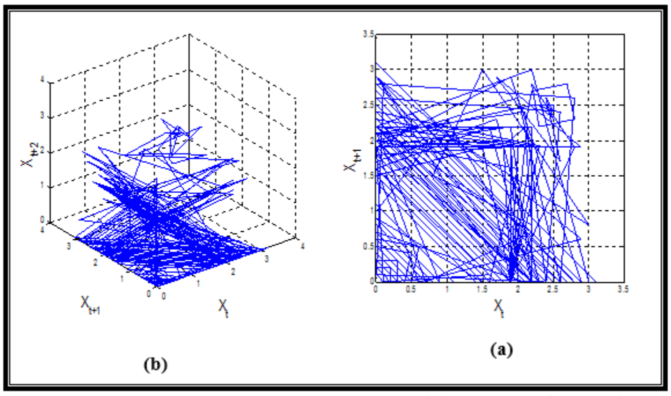


Figure (4): state - space diagram $x(t)$ to $x(t+h)$, $h = 1,2$ Observed of earthquakes in Mosul Dam **(A):** plot two-dimensional state - space diagram. **(B):** state - space diagram in three dimensions.

C. A periodicity

The strongest feature of the statement of chaotic dynamics is the absence of periodic, time series are said to be strictly periodic if $x(t) = x(t+\tau)$ for each t, this to mean the duration of the course $T(>0)$ remains unchanged [13] .

The spectral analysis is good indicators to know the behavior of time series, if the chaotic behavior, the peaks do not appear to be overlapping the top individual, as in cyclical behavior. Figure (5) shows the estimated a function of the density spectrum Observed of earthquakes in Mosul Dam is estimated using the following [8] :

$$f(w) = \frac{1 + 2\sum_{k=1}^{\infty} \hat{\rho}_k \cos(kw_p)}{2\pi}; \quad -2\pi \leq w \leq 2\pi, \ w_p = \frac{2\pi p}{T}; \ p = 1,2,\ldots,\frac{T}{2} \quad ..(20)$$

P is the truncation point, has been selected to be $P = [2\sqrt{T}]$, and that the brackets [] is taken to mean the right .

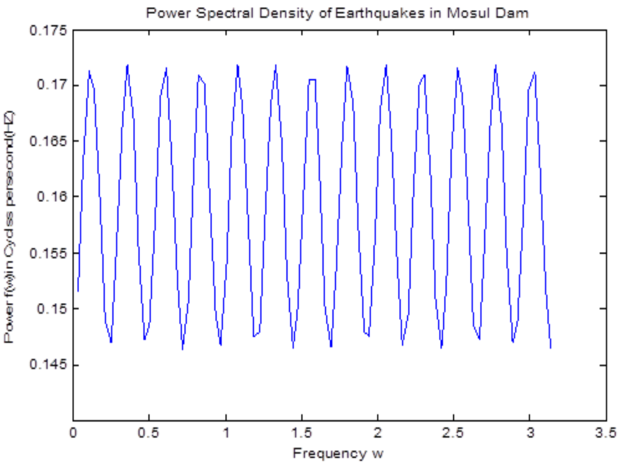


Figure (5) : Estimated power spectral density function observed of earthquakes in Mosul Dam.

Noting the estimated power spectral density function in Figure (5) that the energy in this sequence is distributed in the short term. The absence of a top and a single individual in this spectral function refers to the non-periodic sequence of these, ie non-periodic sequence.

D. Check of the Independence

Observed of earthquakes in Mosul Dam views consisting of 16571 base to determine whether the sequence of independent and have the same probability distribution or not? As not only the examination to diagnose the independence of the sequence, but also used in determining whether to behave chaotic or regular.

As it's using the BDS statistical algorithm index was calculated correlation integral $C(\varepsilon, m)$ defined in equation (4) to observed of earthquakes in Mosul Dam. The results we have obtained is shown in the table (1) .

Table (1): values of the correlation integral of observed of earthquakes in Mosul Dam.. Each column represents Embedding dimension different (m), where $m = 1,2,\ldots.10$, each row shows different values for j, where $\varepsilon_j = (0.5)^j$, each cell represents the value of the correlation integral $C(\varepsilon, m)$ defined in equation (4).

j	(Embedded dimension) m								
	2	3	4	5	6	7	8	9	10
1	0.45841	0.31055	0.21153	0.15372	0.11615	0.088421	0.067929	0.053753	0.043908
2	0.41149	0.26436	0.16859	0.11895	0.089485	0.068191	0.053555	0.043095	0.036268
3	0.35476	0.20774	0.11887	0.077819	0.055698	0.040793	0.031407	0.024733	0.020623
4	0.30422	0.15535	0.073255	0.039893	0.023456	0.013925	0.0084826	0.0051839	0.003555
5	0.27339	0.12866	0.052501	0.026554	0.014834	0.0082116	0.0048836	0.0028106	0.0015895
6	0.27339	0.12866	0.052501	0.026554	0.014834	0.0082116	0.0048836	0.0028106	0.0015895
7	0.27339	0.12866	0.052501	0.026554	0.014834	0.0082116	0.0048836	0.0028106	0.0015895
8	0.27339	0.12866	0.052501	0.026554	0.014834	0.0082116	0.0048836	0.0028106	0.0015895
9	0.27339	0.12866	0.052501	0.026554	0.014834	0.0082116	0.0048836	0.0028106	0.0015895
10	0.27339	0.12866	0.052501	0.026554	0.014834	0.0082116	0.0048836	0.0028106	0.0015895

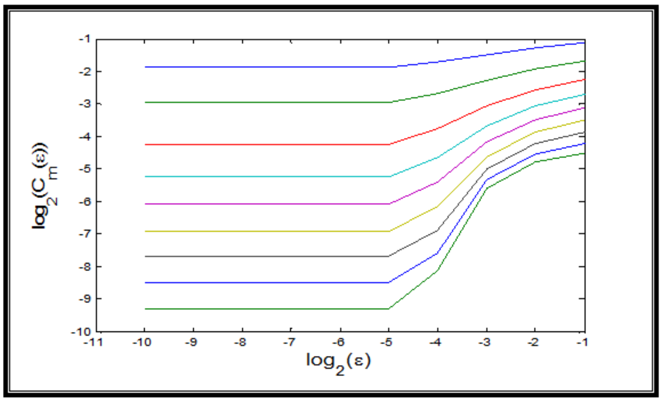


Figure (6): a function $\log_2 C_m(\varepsilon)$ for $\log_2(\varepsilon)$ to the different values of the embedding dimension m for observed of earthquakes in Mosul Dam. Where $\log_2 C_m(\varepsilon)$ versus $\log_2(\varepsilon)$ represented correlation dimension .

by using Algorithm a statistical indicator BDS and after we get the values of the correlation integral of observed of earthquakes in Mosul Dam. in the table (1) is not achieved through the following relationship

$$C_m(\varepsilon) = (C_1(\varepsilon))^m \quad \ldots.(21)$$

Show that the sequence is not independent [4], as well as we can also check the following relationship

$$C_m(\varepsilon)/(C_1(\varepsilon))^m = L \quad \ldots.(22)$$

the following relationship, Where we find the relationship (22) the following: [3]

1- in the case L that the fact that the value of greater than one values in this case be of any strong attraction apart, as they may resemble to some extent the laypanov number, This means a state of chaos.

2- If the value L is less than one, this means there is attraction, but it is tight and there is not any of the state of chaos.

3- When the value L is equal to one definition of chaos can not be achieved and that the variables are independent and identical.

Results obtained from the account of the integration link when different values of the post-submerged in the table (1) we can deduce that the observed of earthquakes in Mosul Dam behave chaotically.

Were also calculate the value of free statistical $W_{m,n}(\varepsilon)$ of the equation (21) to indicate whether the observed of earthquakes in Mosul Dam of independent and have the same probability distribution or not? And the results we have obtained is represented in the table (2).

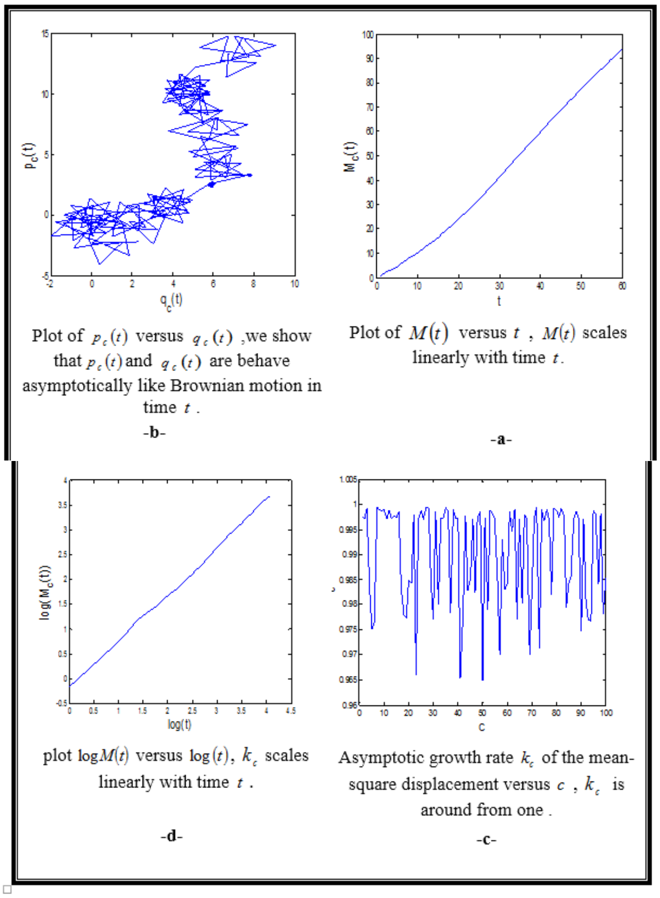
Table (2): values of the free statistic $W_{m,n}(\varepsilon)$ of the observed of earthquakes in Mosul Dam. Each column represents Embedding dimension different (m), where $m = 1,2,\ldots 10$, each row shows different values for j, where $\varepsilon_j = (0.5)^j$, each cell represents the value of the free statistic $W_{m,n}(\varepsilon)$ defined in equation (19).

j	(Embedded dimension) m								
	2	3	4	5	6	7	8	9	10
1	3.5717	2.4025	2.0693	3.1967	4.4341	5.4577	6.3936	7.4744	8.8945
2	4.0149	2.7226	1.9982	3.2347	4.7941	6.1998	7.8244	9.5538	12.012
3	3.1541	1.6533	0.78356	1.7237	2.9967	4.2101	5.7435	7.3438	9.7169
4	2.0557	-0.013535	-1.2151	-0.75251	-0.199397	0.23905	0.63730	0.85540	1.3262
5	1.5295	-0.53442	-1.8798	-1.2844	-0.66646	-0.24763	0.11429	0.299	0.39471
6	1.5295	-0.53442	-1.8798	-1.2844	-0.66646	-0.24763	0.11429	0.299	0.39471
7	1.5295	-0.53442	-1.8798	-1.2844	-0.66646	-0.24763	0.11429	0.299	0.39471
8	1.5295	-0.53442	-1.8798	-1.2844	-0.66646	-0.24763	0.11429	0.299	0.39471
9	1.5295	-0.53442	-1.8798	-1.2844	-0.66646	-0.24763	0.11429	0.299	0.39471
10	1.5295	-0.53442	-1.8798	-1.2844	-0.66646	-0.24763	0.11429	0.299	0.39471

For different values of the level of moral terms α where $\alpha = 0.01$ and $\alpha = 0.05$ reject the null hypothesis Ho: xt is iid, which indicates that the chaotic behavior (not periodic) for a observed of earthquakes in Mosul Dam.

E. The Binary Test for Chaos of the Earthquakes.

The observed of earthquakes in Mosul Dam has difficult to analyze the sequence and determine whether behavior is chaotic or not chaotic standards of chaos traditional but that by the new test ,binary test for chaos, which is characterized by the potential applied to any views dynamically without the need to rebuild the space phase, and when the test was performed bilaterally chaos of the algorithms described in section (2), show that the behavior of observed of earthquakes in Mosul Dam. is chaotic and non-regular terms where $k = 0.998 \cong 1$. as shown in Figure (7) .



Plot of $p_c(t)$ versus $q_c(t)$,we show that $p_c(t)$ and $q_c(t)$ are behave asymptotically like Brownian motion in time t .
-b-

Plot of $M(t)$ versus t , $M(t)$ scales linearly with time t.
-a-

plot $\log M(t)$ versus $\log(t)$, k_c scales linearly with time t .
-d-

Asymptotic growth rate k_c of the mean-square displacement versus c , k_c is around from one .
-c-

Figure(7) : The binary Test for Chaos of the earthquakes in Mosul Dam.

VII. Conclusions

1- Adoption of the binary test, zero – one test of chaos as an effective tool in distinguishing between the chaotic, and regular behavior of non-linear time series , this distinction is very clear by a variable discrimination K value close to either zero or close to one, and when using an algorithm of the binary test and after obtain numerical results and graphs for the observed of earthquakes in Mosul Dam. We get that earthquakes in Mosul Dam behave Chaotically and thus it's behavior can not be predicted in the long-time.

2- The statistical indicator BDS examine the independent of time series data and determine whether they are independent and identically distribution. By use the statistical algorithm BDS on the s observed of earthquakes in Mosul Dam and the creation of the correlation integral values shown in the table (1), and the free statistical values shown in the table (2), it is clear that the observed of earthquakes in Mosul Dam are independent and behave chaotically.

3- Plotting the time series curve, the state space diagram, the spectral function to diagnosis the behavior of the observation of earthquakes in Mosul Dam, these tools and graphic forms (3) and (4) and (5) show that they behave chaotic non-periodic and irregular.

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